Motivational Experiments with Membranes in the Context of Physics Education

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Abstract
This article deals with membranes and their motion. There are full instructions for the preparation of this experiment. One of the most interesting observations is "an unbelievable upward motion of membrane" in the direction opposite to the Gravitational Field. The observations are given in the context of physical laws with explanation and also in the mathematical form. We have introduced a model of the linear harmonic oscillator with membrane, Bernoulli's principle and the Continuity Equation in non-traditional form.

Keywords
Motivational experiment, creativity, motion of membrane, model of harmonic oscillator, continuity equation, Bernoulli's principle.

1 Experiment

Performing experiments is a great way to keep students entertained and to evoke their interest into Science. An experiment should be an inseparable part of the educational process. It is one of the examples of the empirical method. Any experiment has a number of functions, e.g. motivational, heuristic, etc.

Usage of any readily available experiment tools facilitates the development of the creativity. Inquest after explanation of the observed phenomenon can contain elements of humour and detective story. By using mathematical apparatus in the investigation of the physical explanations we can demonstrate important inter discipline relationships. Even a trivial experiment can lead into a thought experiment and compelling results with demonstration of multiple physical effects.

2 Experiment aids and realisation

We will use following tools for the performance of this experiment: a top cut of a plastic bottle, a transparent plastic sleeve, a glue, scissors, a bowl of water with a dish washing fluid and couple of drops of glycerine.

We will need to cut sleeves of two shapes (Figure 1) and glue up the sides to make the shape of rotational cones of different top angles (steepness). We must cut off their tops about 0,5 cm from the top and then dip them into the bowl of the water with the dish washing fluid and several drops of glycerine.

3 Experiment - Observation

When we pick the cones and the top of a plastic bottle out of the liquid, we will observe membranes creation. The membranes will be moving downward with different velocity except for the membrane in the plastic bottle, which will not move.

When we repeat experiment with the objects overturned, the membranes will be moving upwards opposite to the Gravitational Field, except the membrane in the plastic bottle, which stays at rest as before (Figure 2).

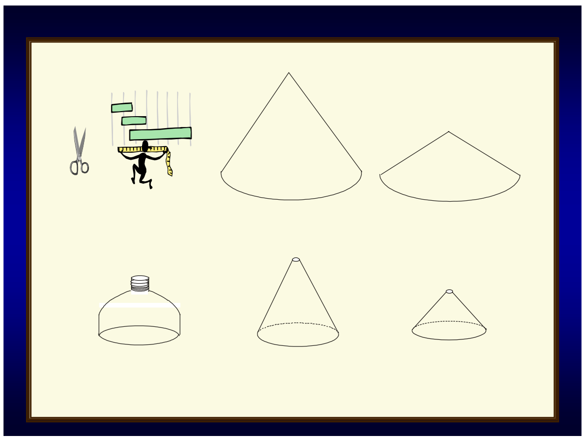

Figure 1

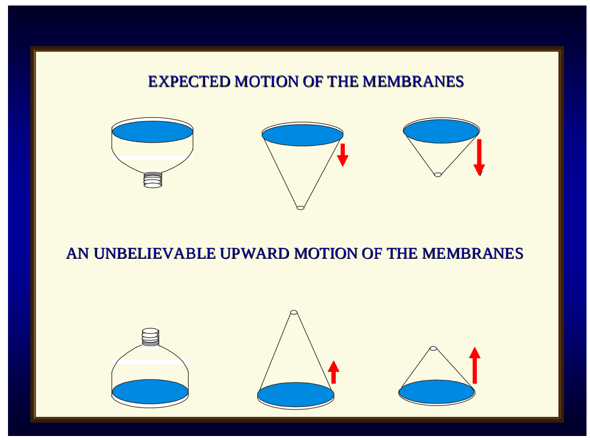

Figure 2

4 Explanations

When we overturn the cone with created membrane then the membrane starts moving. The first reason we might think of why the membrane is moving is due to the Gravitational Field of the Earth. That's, we think, is the reason, why it moves downwards. Let's say that this is the correct theory. In this case, however, how can we explain the other scenario, when the cone is in the original position but the membrane is moving upwards opposite to the Gravitational Force of the Earth? This indicates that our premises were incorrect. We are not going to investigate the impact of the Gravitational Field anymore as a cause of the membrane movement. We can deduce that the Gravitational Field doesn't impact the movement of the membrane in our experiment.

Let's start from the beginning. When we dip and drag out the cone from the liquid there appears a membrane which is different from the water surface. The membrane on our cone has two surfaces – we can imagine it as two layers very close to each other – top and bottom ones. Why this happens? It appears this way because the membrane represents a very thin layer of water bordered by the air.

The behaviour of those two surfaces is similar as the stretched rubber area. The ratio of stretching of the membrane corresponds to the surface tension σ. The value of the surface tension varies for different liquids. For example the value of σ of the water is 73.10^{-3} Nm^{-1}, for the soap water is smaller, approximately 30.10^{-3} Nm^{-1}. The surface tension represents the force F affecting one unit of the length. It means that the surface of the clear water stretches more than the one of the soapy water. That's the reason why there is higher probability of creation of bubbles from the soapy than the clear water.

The membrane has also its so called surface energy which is proportionate to its surface: $E = \sigma.S$. In case of our experiment with the circle membrane with radius R and two surfaces: $E =$

$\sigma.2.S = \sigma.2.\pi R^2$. It means that if the membrane is created from the clear water its energy would be much higher than the one from the soapy water.

If we want to understand the movement of the membrane in the cone we have to answer following question: What quantity is changing when the cone with membrane is moving? In both scenarios of movement - upwards or downwards – the membrane is contracting. Why is this happening? We can describe both situations from energy point of view. We use to describe systems in Physics with associated value or type of energy, e.g. small or big energy, kinetic or potential energy, etc. That's the same with our membrane, which has its energy. When we are stretching the membrane we are performing work which increases the surface tension and consequently the surface energy of the membrane. And vice versa – when the membrane is contracting its surface energy is dropping and the membrane is performing work..

We know that the nature is a very good controller. That's why all the processes in the nature are happening in line with the most economic and energy effective final statuses of systems. Due to this law any surface is contracting to occupy the smallest possible space which corresponds with the most effective energy status.

Minimalization of the energy is the reason why the membrane is moving even against gravitational attraction. The cone allows its membrane to move. Its cross-section is fluently decreasing which made it possible for membrane to copy its surface to contract accordingly. The steeper cone is the easier and faster membrane contracts. In comparision with the shape of the top of the plastic bottle - it is almost flat. Its cross-sections are not decreasing or increasing – we can say that they are fixed. That's the reason why we don't observe any movement of the membrane. This membrane has the same energy values in different places with the same cross-section which would mean that it is stretched with the same surface tension in every place. There is missing reason for its movement due to the difference in the energies.

5 Explanations and Mathematics

Let's have a closer look on our experiment of the moving membrane from the forces point of view. We can introduce some simplifications for easier mathematical calculations. Let's ignore the frictional force between the moving membrane and the surface of the cone. In our experiment we minimise the impact of the frictional force by drenching the cone in before creation of the membrane. Let's assume that the membrane is moving in the environment without any resistance.

In this case we are dealing with two remaining forces. There is a gravitational force of the membrane G and the opposite unknown force F_1 which acts on the membrane in the process of contraction. We will demonstrate that this unknown force F_1 is in proportion with the deviation of the membrane from its state of equilibrium.

The membrane with radius r_1 has its surface energy W_1. That's why the membrane is able to perform work. If the membrane contracts its surface energy decreases. There is the work performed by the membrane in this scenario. It means that there has to act a force F_1 which makes this work. This force moves the membrane upwards as shown in the Figure 3.

We can determine the forces acting on the membrane (let's assume that the weight of the membrane with the thickness of 10^{-6}m is $m=\rho V$=1100kgm$^{-3}\pi$(0,05 m)$^2 10^{-6}$m=8,6.10^{-6}kg). See one special case shown in the Figure 4.

Because $G<<|F_1|$, we can ignore influence of G. Then there is remaining only one force in place which is $F_{1.}$

The force which forces the membrane to move in the second lower cone with high of 0,10 m is higher than F_1 in the taller cone. This force is 4,7.10^{-3} N in this scenario. This means that the speed of movement of membranes in cones with different angles differs. There is a higher speed of the membrane in the lower cone with higher acting force. This is in line with our observation of the experiment.

There is a conclusion that in our idealised scenario the only important force acting on the membrane to consider is F_1. We can modify the mathematical formula as shown in the Figure 4. The equation $F_1=-kx$ reminds us of the formula for expression of the force in harmonic vibration.

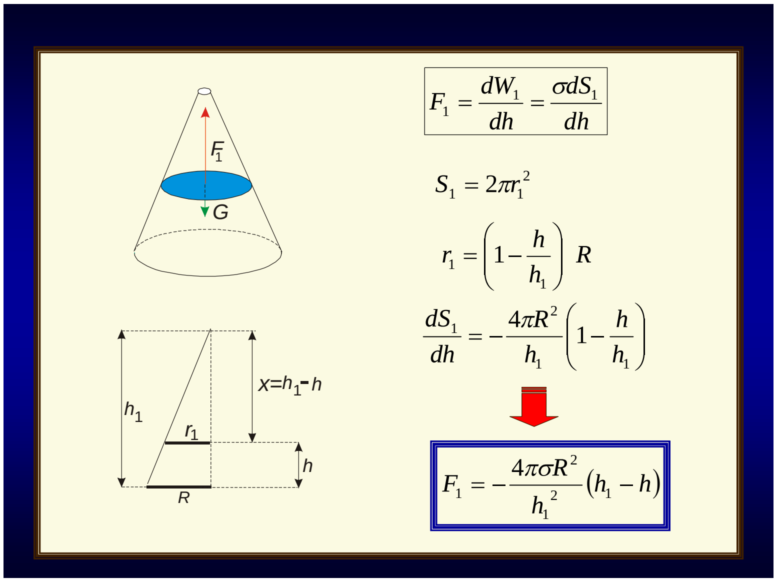

Figure 3

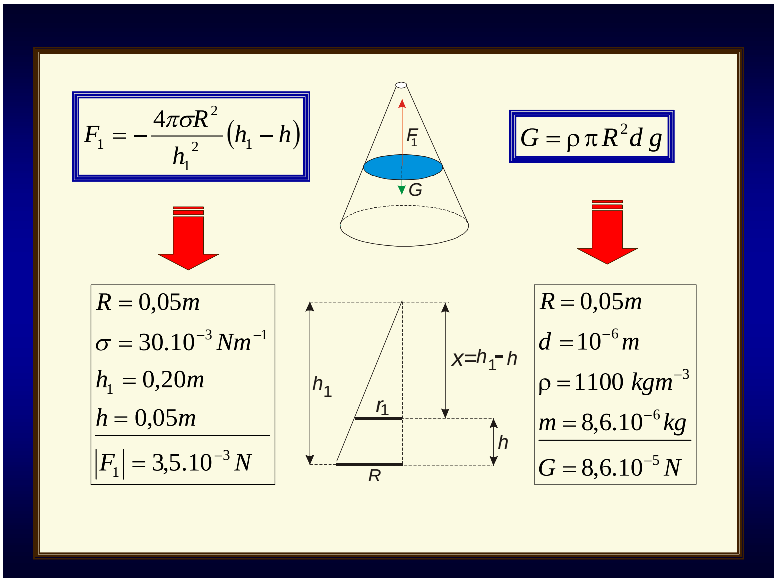

Figure 4

6 Thought experiment- Idealized Oscillations

We can connect two identical cones by their tops leaving a little circle wholes between them to allow the movement of the membrane, as seen in the Figure 5. In the idealised scenario the membrane would contract as moving from its first extreme position and after passing through its equilibrium, which is the connecting point of the cones, it would start stretching on its way to the other extreme position. It would stop while reaching the second extreme position and would start contracting again while moving the opposite direction. The movement would be a periodic one. We can calculate the corresponding time period of this idealised movement as shown in the Figure 5.

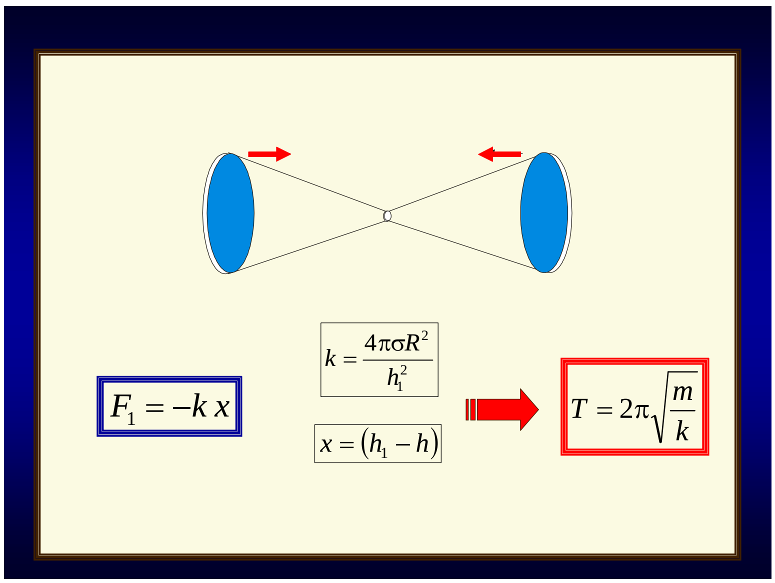

Figure 5

7 The Law of Energy Conservation Point of View

Let's change our point of view on the situation discussed in the paragraph 6 through the Law of the Energy Conservation. We can apply the law on our cone with its radius R and high h_1 as shown in the Figure 6. The initial energy of the created membrane is W_z. Let's consider this position as the base one. We can ignore losses of energy caused by the resistance of the environment and frictions. We know that in the position with high h in the cone the membrane is contracting and loosing its surface energy to the value W. The membrane will gain its kinetic energy E_k.

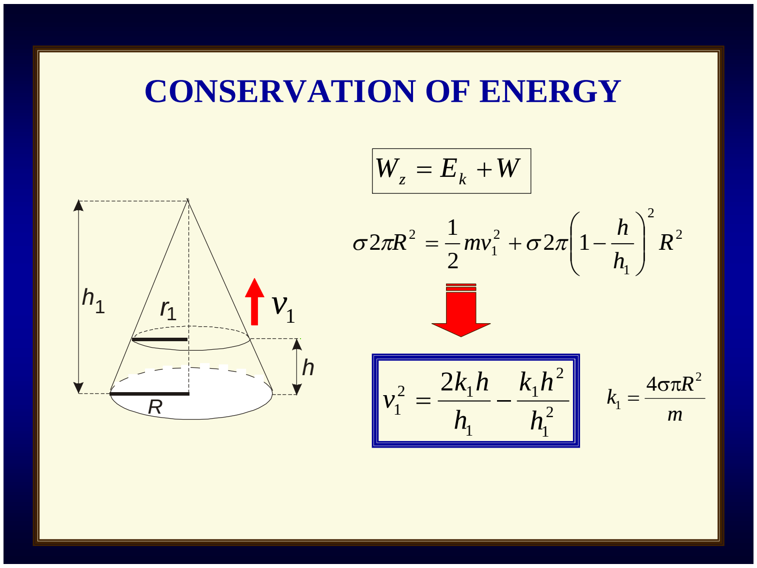

Figure 6

The membrane also increases its potential energy but we can ignore this increase as negligable (you can prove this). We will ignore this energy to simplify our further calculations. The smaller the membrane surface in high h is (W is decreasing) the faster the membrane is moving (E_k is increasing). In line with the Law of Energy Conservation kinetic energy of the membrane is increasing on the account of its surface energy which is decreasing. If the membrane is moving upwards its potential energy is also increasing very slowly continually with increase of high h (negligable increase). However the surface energy decreases with square function and vice versa the kinetic energy increases with square function of the speed.

8 The Speed of the Movement Dependant on the Steepness of the Cone.

We are going to use our calculations and deductions for the movement of the membrane in the cone with high h_1 also for the lower cone with high h_2. We will replace index 1 with index 2 in our previous calculations. We can summarise the conclusion for the movement of the membrane in the cones: There is a dependency between the speeds v_1 and v_2. In our case the first cone is higher than the second, it means $h_1>h_2$, in case of the surface energy of membranes in high h is $W_1>W_2$. The energy in the steeper cone is smaller. We can derive from the Law of Energy Conservation that $E_{k1}<E_{k2}$. It means that the speed of the membrane in the steeper cone is higher, $v_1<v_2$. The taller the cone is (in the sense of steeper) which corresponds to the higher h_1, the slower the movement of the membrane is.

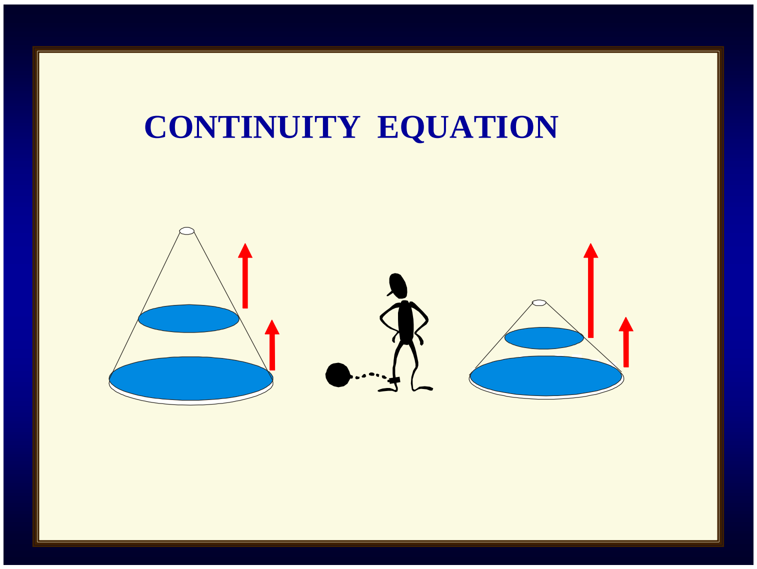

Figure 7

9 Continuity Equations in Non-Traditional Form

If we create two membranes in one cone we will observe different speeds of membranes as shown in the Figure 7. The membrane with the smaller radius will move faster than the membrane with the longer radius. There exists an analogy with Bernoulli's principle. In the case described above the moving fluid is the air confined between those two membranes. The energy of the membranes is not sufficient to compress only to lock the gas up in between. Those two membranes are not moving together. There is a difference in the speed of those membranes. There is not possible that the first membrane would overtake the second one (similar as with trams). The volume of the air is constant regardless of the movement of membranes. Both membranes are trying to decrease their surface which means that they contract and move with the gas trapped in between.

The cone represents a tube continuously changing its cross-section. As we know the water which flows in the tube with different cross-sections, moves slower in the places with bigger diameter while slower in the places with smaller diameter. It works similar way with membranes. We can see the analogy between flow of the liquid and the air trapped and flow between the membranes in our experiment. Our scheme of two membranes simulates the movement of incompressible fluid in the tube with variable cross-sections.

10 Conclusions

One of the ways how to make the education of the physics more popular is experimenting. We showed you the experiment with membranes with more levels of difficulty depending on the targeted group of students. We can start with simple observation then go through the mathematical description of the behaviour of the system and further get into the thought experiment. We have applied the analogy and synthesis of the knowledge. It has leaded us into the model of sustained harmonic oscillator and applied demonstration of the Continuity Equation in the tube with variable cross-sections.

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