Innovative application of Dantzig's North – West Corner Rule to solve a transportation problem

Adwell Mhlanga, Immaculate S. Nduna, Dr Florence Matarise, Albert Machisvo.

Department of Statistics, University of Zimbabwe, Post Office Box MP167, Mount Pleasant,

Harare, ZIMBABWE

Corresponding Author:

Adwel Mhlanga
Department of Statistics,
University of Zimbabwe,
Post Office Box MP167,
Mount Pleasant, Harare,
ZIMBABWE

Phone: 303211 Extensions 15058 or 15059

Cell No. 0712 556 950

0776 918 011

email: mhlanga49@gmail.com

Abstract

The North – West Corner Rule for obtaining an initial basic feasible solution to a transportation problem disregards costs. This disregard necessarily leads to a high number of iterations to optimality.

A search for a way to incorporate costs in the use of the North-West Corner Rule led to experimenting with the creation of a north-west south-east low cost broad band orientation in the transportation cost matrix. The transformed table was tested by applying the North-West Corner Rule on it. The findings were very encouraging though not conclusive because of the small sample size. Practitioners may want to adopt this innovation to confirm its effectiveness.

Key words: basic, feasible, initial, innovative, optimal, solution.

1. Introduction

In its simplest form the transportation problem is that of determining the least cost schedule of transporting a homogenous commodity from m sources to n destinations. Manually, the problem is solved in two stages. First an initial basic feasible solution is obtained. In the second stage some iterations are performed until an optimal schedule is achieved. With a few exceptions the various manual methods for solving the problem differ in the way the initial basic feasible solution is obtained. The computations towards optimality use the same method.

Currently the North-West Corner Rule for obtaining the initial solution is the least efficient method; the method leads to a relatively very high number of iterations to optimality. In this paper an innovative method for applying the North-West Corner Rule is introduced and its impact on the efficiency of the method is tested.

2. Literature Review

The transportation problem was first stated by Hitchcock (1941). The solution to this problem was first offered by George B. Dantzig and published in Koopman's (1951) monograph 13. The procedure that Dantzig used to obtain an initial basic feasible solution was later termed the North-West Corner Rule by Charmes and Cooper (1954 to 1955). Dantzig's initial solution ignored costs. But he developed an iterative procedure for computing the optimal solution which is still in use today. He considered that the optimal solution would be obtained in at most $m + n - 1$ iterations where m is the number of sources and n is the number of destinations.

Several practitioners have developed alternative methods for determining an initial basic feasible solution which takes costs into account. Their methods are considered in this article. The most popular of these are the methods by Vogel (1958) and Russell (1961). These two methods as well as the other methods that will be applied in this article use Dantzig's procedure to move the solution to optimality.

Assume that *m* sources have a_i units of a homogenous commodity that is to be transported to n destinations which require b_j units of the commodity. Let c_{ij} be the unit cost of transporting a commodity from source *i* to destination *j*, and let x_{ij} units be associated with the cost c_{ij}. Then the total cost of transporting x_{ij} units from source *i* to destination *j* is $c_{ij}x_{ij}$. The cost of transporting the entire commodity is to be minimized. The problem formulation is the following.

Minimize $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$

such that $\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1,2,...,m,$ (1)

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1,2,...,n, \qquad (2)$$

and $x_{ij} \geq 0.$ (3)

Generally this problem has a solution If

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} = \sum_{j=1}^{n}\sum_{i=1}^{m} x_{ij}, \quad (4)$$

that is, if $$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \quad (5)$$

The problem can be solved as a linear programming problem with $m+n$ equations in mn variables. If we add up the n equations in (2) and subtract the sum of any *m – 1* equations in (1) we get a result that equals the remaining equation in (1). This means that this one remaining equation is redundant in the system of *m+n* equations and so can be eliminated. Therefore we have only *m+n - 1* independent equations in mn unknowns. Solving these equations algebraically would give us at most *m+n -1* positive values x_{ij} together with at least *mn* - (*m+n -1*) value $x_{ij} = 0$.

The transportation problem is solved in two stages. First, an initial basic feasible solution is obtained using any of the methods described below. In the second stage at least one optimality test is conducted. If the current solution is not optimal a computation that improves the solution is done. This procedure is repeated until an optimal solution is obtained.

The methods for finding an initial basic feasible solution are well documented. They are summarized below. Before any method is applied the problem is reduced to a rectangular array of *m* by *n* unit costs c_{ij}, a right margin column of supplies a_i and a bottom margin row of demands b_j.

Using the North-West Corner Rule method, an amount x_{11} units of the commodity is allocated at cell (*1*,*1*) in such a way that a_1 is exhausted or b_1 is exhausted. If $x_{11} = a_1$, row *1* is crossed and an allocation of an amount $x_{21} = \min(a_2, b_1 - a_1)$ is made at cell (*2*,*1*). If $x_{11} = b_1$ column *1* is crossed and an allocation of an amount $x_{12} = \min(a_1 - b_1, b_2)$ is made at cell (*1*,*2*). If $x_{11} = a_1 = b_1$, either the row or the column is crossed and a reduced demand or supply of 0 is entered in the margin of the uncrossed column or row. Thus $a_1 - b_1$ *or* $b_1 - a_1$ may be 0. Subsequent allocations are made using the same logic. The last allocation is made at cell (*m, n*). Altogether there will be *m* + *n -1* {= (*m -1*) + *n* or *m* + (*n -1*)} allocations.

Using Vogel's approximation method, for each row and each column of the table the difference between the lowest cost and the next lowest cost is computed and recorded under a penalty column or penalty row in the margins. Then in the row or column with the highest penalty an allocation of x_{ij} units of the commodity is placed in the cell which has the lowest cost c_{ij} where x_{ij} exhausts a_i supply or b_j demand. If a_i units are exhausted row *i* is crossed. Demand b_j is reduced to $b_j - a_i$ and column penalties are updated. If b_j units are exhausted column *j* is crossed. Demand a_i is reduced to $a_i - b_j$ and row penalties are updated. Ties are broken arbitrarily. The process is repeated until the allocation process is complete.

Using Russell's approximation method, for each row the highest cost c_{ij} is identified and recorded on the right margin under $\bar{u}_i$. For each column the highest cost c_{ij} is identified and recorded on the bottom margin under $\bar{v}_j$. Then for each cell (*i,j*), $\bar{u}_i + \bar{v}_j - c_{ij}$ is computed and the result entered in cell (*i,j*) and encircled. After the process is completed, an allocation x_{ij} of the commodity is made in the cell with the highest encircled value. If $x_{ij} = a_i$ row *i* is crossed, the demand b_j is reduced to $b_j - a_i$, the highest remaining c_{ij} in each column is identified and $\bar{v}_j$'s updated. If instead $x_{ij} = b_j$ is allocated to cell (*i,j*), column *j* is crossed, a_i is reduced to $a_i - b_j$, the highest remaining c_{ij} in each row is identified and $\bar{u}_i$'s updated. Ties are broken arbitrarily. The process is repeated until the allocation process is completed.

Using the Least Cost (also called Matrix Minima) method the smallest c_{ij} in the table is identified and an amount x_{ij} allocated at cell (*i,j*) If $x_{ij} = a_i$, row *i* is crossed and b_j is reduced to $b_j - a_i$. If instead $x_{ij} = b_j$ column *j* is crossed and a_i reduced to $a_i - b_j$. Ties are broken arbitrarily. The smallest remaining c_{ij} is identified and the process repeated until the allocation process is complete.

In the Row Minima method, the smallest cost c_{1j} in row 1 determined and an allocation x_{ij} made at cell (*1,j*). If $x_{1j} = a_1$, the first row is crossed, demand b_j is reduced to $b_j - a_1$ and row 2 considered. If $x_{1j} = b_j$, column *j* is crossed and a_1 is reduced to $a_1 - b_j$. In this case the remaining lowest cost c_{1j} is identified and an allocation made at the corresponding cell. Ties are broken arbitrarily. The process is continued until all the allocations are made.

In the Column Minima method, the smallest cost c_{i1} is identified and an amount x_{i1} allocated to cell (*i*,*1*). If $x_{i1} = a_i$, row *i* is crossed and b_1 reduced to $b_1 - a_i$. Column 1 is reconsidered and the process is repeated with the remaining columns. If instead $x_{i1} = b_1$, column 1 is crossed and a_i reduced to $a_i - b_1$. In this case the smallest remaining cost c_{i2} is identified and an allocation made at cell (*i*,*2*). Ties are broken arbitrarily. The process is continued until the allocation is complete.

The above methods for obtaining an initial basic feasible solution have been summarized from Gass, S.I. (2010), Gupta and Man Mohan (1974), Hillier and Lieberman (1995), Imam et. al. (2009) and Taha (2008). The procedure for computations to optimality is the same; therefore it is not being considered here.

Since the North-West Corner Rule method disregards the costs c_{ij}: its initial solution has a high cost. Consequently, the number of iterations to optimality is usually high. Vogel's and Russell's approximation methods provide the least number of iterations to optimality. For these two methods sometimes the initial basic feasible solution is optimal.

Many articles have been published in which Vogel's approximation method has been investigated for improvement. These include articles by Shimshak, Kaslik and Barclay (1981), Mathirajan and Meenaksli (2004), and Korukoglin and Balli (2011). However ways of improving the efficiency of the North-West Corner Rule method by considering costs have not been reported.

3. Materials and Methods

Four sample transportation problems were selected at random from Gupta and Man Mohan, Hillier and Lieberman, and Taha. The costs c_{ij}, supplies a_i demands b_j are given in tables 1a, 2a, 3a, and 4a. Using these tables, the North-West Corner Rule, Vogel's approximation, Russell's approximation, Least Cost, Row Minima, and Column Minima methods were used to find the initial basic feasible solution. Next, tables 1a, 2a, 3a, and 4a were innovatively transformed to tables 1b, 2b, 3b and 4b respectively, through row or column manipulation. The manipulation was designed to create tables with low cost north-west south-east broad band orientations. The North-West Corner Rule method was then applied to test the innovation's efficiency. For each method used, optimality tests were conducted and where necessary further computations were made until optimal allocations were obtained.

4. Results and Discussion

The number of iterations to optimality for each problem and each method are shown in table 5. The table shows that innovative North - West Corner Rule method is second only to Vogel's Approximation method out of the seven methods considered. Therefore an informed and imaginative manipulation of the rows or columns of the transportation cost matrix makes the North – West Corner Rule method quite efficient.

5. Conclusion and Recommendation

Since the North-West Corner Rule is simple to understand and to apply, it is popular. But the large number of iteractions disadvantages it. It is recommended to intintively manipulate the rows or columns before applying the method. Many more examples should be solved to establish the superiority of the new method to the other methods apart from Vogel's.

Table 1a. Cost, supply and demand figures for sample problem 1.

		Destination				
		1	2	3	4	Supply
Source	1	1	2	1	4	30
	2	4	2	5	9	50
	3	20	40	30	10	20
	Demand	20	40	30	10	

Table 1b. A reproduction of table 1a figures but with columns 1 and 3 interchanged.

		Destination				
		3	2	1	4	Supply
	1	1	2	1	4	30
Source	2	5	2	4	9	50
	3	30	40	20	10	20
	Demand	30	40	20	10	

Table 2a. Cost, supply and demand figures for sample problem 2.

		Destination				
		1	2	3	4	Supply
	1	21	16	25	13	11
Source	2	17	18	14	23	13
	3	32	27	18	41	19
	Demand	6	10	12	15	

Table 2b. A reproduction of table 2a figures but with column displacement.

		Destination				
		4	1	2	3	Supply
	1	13	21	16	25	11
Source	2	23	17	18	14	13
	3	41	32	27	18	19
	Demand	15	6	10	12	

Table 3a. Cost, supply and demand figures for sample problem 3.

		Destination				
		1	2	3	4	Supply
	1	1	4	6	5	14
Source	2	6	3	4	2	5
	3	2	9	8	7	16
	Demand	15	10	6	4	

Table 3b. A reproduction of table 3a figures but with rows 2 and 3 interchanged.

		Destination				
		1	2	3	4	Supply
	1	1	4	6	5	14
Source	3	2	9	8	7	16
	2	6	3	4	2	5
	Demand	15	10	6	4	

Table 4a. Cost, supply and demand figures for sample problem 4.

		Destination					
		1	2	3	4	5	Supply
	1	4	9	8	10	12	24
Source	2	6	10	3	2	3	18
	3	3	2	7	10	3	20
	4	3	5	5	4	8	16
	Demand	10	20	10	18	20	

Table 4b. A reproduction of table 4a figures with rows displaced.

		Destination					
		1	2	3	4	5	Supply
	3	3	2	7	10	3	20
Source	1	4	9	8	10	12	24
	4	3	5	5	4	8	16
	2	6	10	3	2	3	18
	Demand	10	20	10	18	20	

Figure 1. Solutions to problem 1

Initial allocation							Total Cost	Status
NWCR	$x_{11} = 20$	$x_{12} = 10$	$x_{22} = 30$	$x_{23} = 20$	$x_{33} = 10$	$x_{34} = 10$	600	not optimal
INWCR/Vogel's	$x_{13} = 30$	$x_{21} = 10$	$x_{22} = 40$	$x_{31} = 10$	$x_{34} = 10$	$(x_{23}/x_{11} = 0)$[1]	450	Optimal
Russell's	$x_{11} = 10$	$x_{13} = 20$	$x_{22} = 40$	$x_{23} = 10$	$x_{31} = 10$	$x_{34} = 10$	460	not optimal
LC/RM	$x_{11} = 20$	$x_{13} = 10$	$x_{22} = 40$	$x_{23} = 10$	$x_{33} = 10$	$x_{34} = 10$	560	not optimal
CM	$x_{11} = 20$	$x_{13} = 10$	$x_{22} = 30$	$x_{23} = 20$	$x_{32} = 10$	$x_{34} = 10$	650	not optimal

Second allocation Total cost Status

NWCR	$x_{11} = 10$	$x_{12} = 20$	$x_{22} = 20$	$x_{23} = 30$	$x_{31} = 10$	$x_{34} = 10$	540	not optimal
Russell's	$x_{11} = 0$	$x_{13} = 30$	$x_{21} = 10$	$x_{22} = 40$	$x_{31} = 10$	$x_{34} = 10$	450	Optimal
LC/RM/CM	$x_{11} = 10$	$x_{13} = 20$	$x_{22} = 40$	$x_{23} = 10$	$x_{31} = 10$	$x_{34} = 10$	460	not optimal

Third allocation Total cost Status

NWCR	$x_{11} = 10$	$x_{13} = 20$	$x_{22} = 40$	$x_{23} = 10$	$x_{31} = 10$	$x_{34} = 10$	460	not optimal
LC/RM/CM	$x_{11} = 0$	$x_{13} = 30$	$x_{21} = 10$	$x_{22} = 40$	$x_{31} = 10$	$x_{34} = 10$	450	Optimal

Fourth allocation Total cost Status

NWCR	$x_{11} = 0$	$x_{13} = 30$	$x_{21} = 10$	$x_{22} = 40$	$x_{31} = 10$	$x_{34} = 10$	450	Optimal

Note 1: $X_{23} = 0$ for INWCR, and $x_{11} = 0$ for Vogel's method.

Solutions to problem 2

Initial allocation Total cost Status

NWCR	$x_{11} = 6$	$x_{12} = 5$	$x_{22} = 5$	$x_{23} = 8$	$x_{33} = 4$	$x_{34} = 5$	1095	not optimal
INWCR/Vogel's Russell's	$x_{14} = 11$	$x_{21} = 6$	$x_{22} = 3$	$x_{24} = 4$	$x_{32} = 7$	$x_{33} = 12$	796	optimal
LM/RM	$x_{14} = 11$	$x_{21} = 1$	$x_{23} = 12$	$x_{31} = 5$	$x_{32} = 10$	$x_{34} = 4$	922	not optimal
CM	$x_{12} = 10$	$x_{14} = 1$	$x_{21} = 6$	$x_{23} = 7$	$x_{33} = 5$	$x_{34} = 14$	1037	not optimal

Second allocation Total cost Status

NWCR	$x_{11} = 6$	$x_{14} = 5$	$x_{22} = 10$	$x_{23} = 3$	$x_{33} = 9$	$x_{34} = 10$	985	not optimal
LC/RM	$x_{14} = 11$	$X_{21} = 6$	$x_{23} = 7$	$x_{32} = 10$	$x_{33} = 5$	$x_{34} = 4$	867	not optimal
CM	$x_{12} = 3$	$X_{14} = 8$	$x_{21} = 6$	$x_{22} = 7$	$x_{33} = 12$	$x_{34} = 7$	883	not optimal

Third allocation Total cost Status

NWCR	$x_{11} = 3$	$x_{14} = 8$	$x_{21} = 3$	$x_{22} = 10$	$x_{33} = 12$	$x_{34} = 7$	901	not optimal
LC/RM	$x_{14} = 11$	$x_{12} = 6$	$x_{23} = 3$	$x_{24} = 4$	$x_{32} = 10$	$x_{33} = 9$	811	not optimal
CM	$x_{14} = 11$	$x_{21} = 6$	$x_{22} = 7$	$x_{32} = 3$	$x_{33} = 12$	$x_{34} = 4$	832	not optimal

Fourth allocation Total cost Status

NWCR	$x_{14} = 11$	$x_{21} = 6$	$x_{22} = 7$	$x_{32} = 3$	$x_{33} = 12$	$x_{34} = 4$	832	not optimal
LC/RM/CM	$x_{14} = 11$	$x_{21} = 6$	$x_{22} = 3$	$x_{24} = 4$	$x_{32} = 7$	$x_{33} = 12$	796	optimal

Fifth allocation Total cost Status

NWCR	$x_{14} = 11$	$x_{21} = 6$	$x_{22} = 3$	$x_{24} = 4$	$x_{32} = 7$	$x_{33} = 12$	796	optimal

Solutions to problem 3

Initial allocation Total cost Status

NWCR	$x_{11} = 14$	$x_{21} = 1$	$x_{22} = 4$	$x_{32} = 6$	$x_{33} = 6$	$x_{34} = 4$	162	not optimal
INWCR	$x_{11} = 14$	$x_{23} = 1$	$x_{24} = 4$	$x_{31} = 1$	$x_{32} = 10$	$x_{33} = 5$	158	not optimal
Vogel's	$x_{12} = 10$	$x_{13} = 4$	$x_{23} = 1$	$x_{24} = 4$	$x_{31} = 15$	$x_{33} = 1$	114	optimal
Russell's	$x_{12} = 5$	$x_{13} = 6$	$x_{14} = 3$	$x_{22} = 5$	$x_{31} = 15$	$x_{34} = 1$	123	not optimal
LC/RM/CM	$x_{11} = 14$	$X_{22} = 1$	$x_{24} = 4$	$x_{31} = 1$	$x_{32} = 9$	$x_{33} = 6$	156	not optimal

Second allocation Total cost Status

NWCR	$x_{11} = 14$	$x_{22} = 5$	$\mathbf{x_{31} = 1}$	$x_{32} = 5$	$x_{33} = 6$	$x_{34} = 5$	152	not optimal
INWCR	$x_{11} = 4$	$x_{12} = 10$	$x_{23} = 1$	$x_{24} = 4$	$x_{31} = 11$	$x_{33} = 5$	118	not optimal
Russell's	$x_{12} = 8$	$x_{13} = 6$	$x_{22} = 2$	$x_{24} = 3$	$x_{31} = 15$	$x_{34} = 1$	117	not optimal
LC/RM/CM	$x_{11} = 5$	$x_{12} = 9$	$x_{22} = 1$	$x_{24} = 4$	$x_{31} = 10$	$x_{33} = 6$	120	not optimal

Third allocation Total cost Status

NWCR	$x_{11} = 5$	$x_{12} = 9$	$x_{22} = 1$	$x_{24} = 4$	$x_{31} = 10$	$x_{33} = 6$	120	not optimal
INWCR	$x_{12} = 10$	$x_{13} = 4$	$x_{23} = 1$	$x_{24} = 4$	$x_{31} = 15$	$x_{33} = 1$	114	not optimal
Russell's	$x_{12} = 9$	$x_{13} = 5$	$x_{22} = 1$	$x_{24} = 4$	$x_{31} = 15$	$x_{33} = 1$	115	not optimal
LC/RM/CM	$x_{11} = 4$	$x_{12} = 10$	$x_{23} = 1$	$x_{24} = 4$	$x_{31} = 11$	$x_{33} = 5$	118	not optimal

Fourth allocation Total cost Status

NWCR	$x_{11} = 5$	$x_{12} = 9$	$x_{22} = 1$	$x_{24} = 4$	$x_{31} = 10$	$x_{33} = 6$	120	not optimal
Russell's LC/RM/CM	$x_{12} = 10$	$x_{13} = 4$	$x_{23} = 1$	$x_{24} = 4$	$x_{31} = 15$	$x_{33} = 1$	114	optimal

Fifth allocation Total cost Status

NWCR	$x_{11} = 4$	$x_{12} = 10$	$x_{23} = 1$	$x_{24} = 4$	$x_{31} = 11$	$x_{33} = 5$	118	not optimal

Sixth allocation Total cost Status

NWCR	$x_{12} = 10$	$x_{13} = 4$	$x_{23} = 1$	$x_{24} = 4$	$x_{31} = 15$	$x_{33} = 1$	114	Optimal

Solutions to problem 4

Initial allocation Total cost Status

NWCR	$x_{11} = 10$	$x_{12} = 14$	$x_{22} = 6$	$x_{23} = 10$	$x_{24} = 2$	$x_{34} = 16$	$x_{35} = 4$	$x_{45} = 16$	560	not optimal
INWCR	$x_{12} = 10$	$x_{13} = 10$	$x_{14} = 4$	$x_{25} = 18$	$x_{31} = 10$	$x_{32} = 10$	$x_{44} = 14$	$x_{45} = 2$	386	not optimal
Vogel's	$x_{11} = 10$	$x_{13} = 10$	$x_{14} = 2$	$x_{15} = 2$	$x_{35} = 18$	$x_{32} = 20$	$x_{35} = 0$	$x_{44} = 16$	322	not optimal
Russell's	$x_{11} = 10$	$x_{12} = 2$	$x_{13} = 10$	$x_{14} = 2$	$x_{25} = 18$	$x_{32} = 18$	$x_{35} = 2$	$x_{44} = 16$	318	not optimal
LC	$x_{13} = 4$	$x_{15} = 20$	$x_{24} = 18$	$x_{25} = 0$	$x_{25} = 20$	$x_{35} = 0$	$x_{41} = 10$	$x_{43} = 6$	408	not optimal
RM	$x_{11} = 10$	$x_{13} = 10$	$x_{14} = 4$	$x_{24} = 14$	$x_{25} = 4$	$x_{32} = 20$	$x_{35} = 0$	$x_{45} = 16$	368	not optional
CM	$x_{14} = 4$	$x_{15} = 20$	$x_{23} = 10$	$x_{24} = 8$	$x_{31} = 10$	$x_{32} = 10$	$x_{42} = 10$	$x_{44} = 6$	450	not optional

Second allocation Total cost Status

NWCR	$x_{11} = 10$	$x_{22} = 14$	$x_{23} = 10$	$x_{24} = 8$	$x_{34} = 10$	$x_{35} = 10$	$x_{42} = 6$	$x_{45} = 10$	452	not optimal
INWCR/RM	$x_{11} = 10$	$x_{13} = 10$	$x_{14} = 4$	$x_{25} = 18$	$x_{31} = 0$	$x_{32} = 20$	$x_{44} = 14$	$x_{45} = 2$	326	not optimal
Vogel's	$x_{11} = 10$	$x_{12} = 2$	$x_{13} = 10$	$x_{14} = 2$	$X_{25} = 18$	$x_{32} = 18$	$x_{35} = 2$	$x_{44} = 16$	318	not optimal
Russell's	$x_{11} = 10$	$x_{12} = 4$	$x_{13} = 10$	$x_{24} = 2$	$x_{25} = 16$	$x_{32} = 16$	$x_{35} = 4$	$x_{44} = 16$	316	not optimal
LC	$x_{13} = 10$	$x_{15} = 14$	$x_{24} = 12$	$x_{25} = 6$	$x_{32} = 20$	$x_{35} = 0$	$x_{41} = 10$	$x_{44} = 6$	384	not optimal
CM	$x_{11} = 4$	$x_{15} = 20$	$x_{23} = 10$	$x_{24} = 8$	$x_{31} = 6$	$x_{32} = 14$	$x_{42} = 6$	$x_{44} = 10$	418	not optimal

Third allocation Total cost Status

NWCR	$x_{11} = 10$	$X_{12} = 4$	$x_{13} = 10$	$x_{23} = 0$	$x_{34} = 18$	$x_{35} = 20$	$x_{42} = 16$	$x_{45} = 0$	332	not optimal
INWCR/RM	$x_{11} = 10$	$x_{12} = 2$	$x_{14} = 10$	$x_{14} = 2$	$x_{35} = 18$	$x_{32} = 18$	$x_{35} = 2$	$x_{44} = 16$	318	not optimal
Vogel's	$x_{11} = 10$	$X_{12} = 4$	$x_{13} = 10$	$x_{24} = 2$	$x_{25} = 16$	$x_{32} = 16$	$x_{35} = 4$	$x_{44} = 16$	316	optimal
LC	$x_{11} = 10$	$x_{13} = 10$	$x_{15} = 4$	$x_{24} = 2$	$x_{25} = 16$	$x_{32} = 20$	$x_{35} = 0$	$x_{44} = 16$	324	not optimal
CM	$x_{11} = 10$	$x_{15} = 14$	$x_{23} = 10$	$x_{24} = 2$	$x_{25} = 6$	$x_{31} = 0$	$x_{32} = 20$	$x_{44} = 16$	364	not optimal

Fourth allocation Total cost Status

NWCR	$x_{11} = 10$	$x_{12} = 4$	$x_{13} = 10$	$x_{24} = 18$	$x_{25} = 0$	$x_{35} = 20$	$x_{42} = 16$	$x_{45} = 0$	332	not optimal
INWCR/LC/RM	$x_{11} = 10$	$x_{12} = 4$	$x_{13} = 8$	$x_{24} = 2$	$x_{25} = 16$	$x_{32} = 16$	$x_{35} = 4$	$x_{44} = 16$	316	optimal
CM	$x_{11} = 10$	$x_{15} = 14$	$x_{23} = 10$	$x_{24} = 2$	$x_{25} = 6$	$x_{32} = 20$	$x_{35} = 0$	$x_{44} = 16$	364	not optimal

Fifth allocation Total cost Status

NWCR	$x_{11} = 10$	$x_{12} = 4$	$x_{13} = 10$	$x_{24} = 18$	$x_{25} = 0$	$X_{35} = 20$	$x_{42} = 16$	$x_{44} = 0$	332	not optimal
CM	$x_{11} = 10$	$x_{13} = 10$	$X_{15} = 4$	$x_{24} = 2$	$x_{25} = 16$	$x_{32} = 20$	$x_{35} = 0$	$x_{44} = 16$	324	not optimal

Sixth allocation Total cost Status

INWCR/CM	$x_{11} = 10$	$x_{12} = 4$	$x_{13} = 10$	$x_{24} = 2$	$x_{25} = 16$	$x_{32} = 16$	$x_{35} = 4$	$x_{44} = 16$	316	optimal

Table 5: Number of iterations to optimality for each problem and each method.

Problem	NWCR	INWCR	Vogel's	Russell's	LC	RM	CM
1	4	1	1	2	3	3	3
2	5	1	1	1	4	4	4
3	6	3	1	4	4	4	4
4	6	4	3	2	4	4	6

Legend: NWCR – North – West Corner Rule

INWCR - Innovative North – West Corner Rule

LC – Least Cost

RM – Row Minima

CM – Column Minima

References

Balakrishnan, N., 1990. Modified Vogel's approximation method for unbalanced transportation problem. *Applied Mathematics Letters 3 (2), 9 -11.*

Charnes A. & Cooper, W. W., 1954 – 1955. The stepping stone method of explaining linear programming calculations in transportation problems. *Management Science,* volume *1.*

Dantzig, G. B., 1951. Application of the Simplex Method to a Transportation Problem. *Chapter 23, 359 to 373, of Coopmans [8].*

Gass, I. S., 2010. Linear Programming Methods and Applications. *Fifth Edition*, *Dover Publications*.

Gupta, P. K. & Man Mohan, 1974. Linear Programming. *Marwah Book Agency.*

Hillier, F. S. & Lieberman, G. J., 2000. Introduction to Operations Research. *Seventh Edition, McGraw Hill.*

Hitchcock, F. L., 1941. *Distribution of a product from several sources to numerous localities*. Journal of Mathematical Physics, Vol. 20, 224 - 230.

Imam, T., Elsharawy, G., Gomah, M., & Samy, I (2009). Solving transportation problem using object – oriented model. IJCSNS International Journal of Computer Science and Network Security.

Koopman, T. C., 1951. *Activity analysis of production and allocation*. Cowless Commission Monogram 13, John Wiley and Sons, New York.

Korikoglin, S. & Balli, S., 2011. *An improved Vogel's approximation method for the transportation problem*. Mathematical and Computational Applications, Vol. 16, No.2, 370 – 381.

Mathirajan, M. & Menaksli, B., 2004. *Exponential analysis of some variants of Vogel's approximation method*. Asia – Pacific Journal of Operations Research 2, (4) 447 – 462.

Ramakrishnan, G. S., 1988. *An improvement of Goyal's modified VAM for the unbalanced transportation problem*. Journal of Operations Research Society 39(6), 609 – 610

Russell's E. J., 1969*. Extension of Dantzig's algorithm to finding an initial near – optimal basis for the transportation problem*. Operations Research, 17: 187 – 191.

Shimshak, D. G., Kaslik, J. A., & Barclay, T. D., 1981. *A Modification of Vogel's approximation method through the use of heuristics.* Infor 19, 259 – 263.

Taha, H.A. 2008. Operations Research: An Introduction. *Eighth Edition, Prentice Hall*.