On the cultivation of mathematical ability in the example teaching

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Abstract: The mathematical ability is a special kind of mental ability, and the new curriculum reform emphasizes that teaching should mainly cultivate the students' mathematical ability. As an important part of the mathematics textbook, the example teaching should pay attention to the students' observation, induction, association, conversion, modeling in mathematics and other mathematical ability .
Key words: Example teaching; Mathematical ability; Cultivate

Mathematical example is an important part of mathematics textbooks, and the example teaching is an essential link of the mathematics classroom teaching. The example not only consolidate the basic concepts, but also cultivate students' ability of applying knowledge. Intensifying analysis on the examples in the textbook could help students fully understand the knowledge, seize the point, break through the difficulty, refine thought, cultivate ability, and promote the comprehensive development of students' knowledge, thinking and ability. Based on the high school mathematics textbook and combined with concrete examples, this paper mainly talks about the cultivation of mathematical ability in the example teaching.

1. Mathematical Observation Ability

Observation is necessary and the first step in the process of mathematical thinking. Many excellent discoveries come from the observation and all the mathematical activities begin with observation. Only through careful observation, can we carry out the correct and effective mathematical activities and get the corresponding mathematical conclusions.

Case one: Write down a formula of the following sequences , making its the first four items are the following:

(1) $1,-\frac{1}{2},\frac{1}{3},-\frac{1}{4}$;

(2) $2,0,2,0$

The above case is the first example of the first section C*oncept of Sequence and Simple Representation* on Chapter 2 *The Sequence* of the high school mathematics compulsory textbook-5.This example requests student to write out a formula of the two series in the question on the basis of learning the basic concepts and simple representation of sequence. Through the previous study, we all know the formula is suitable for expressing each number in the series, that is to say the finally written formula is general. Observing the two series, first of all, it's not hard to find out the first four items' absolute value are the reciprocal of the serial number in first sequence , and the odd terms are positive, even terms are negative; The first four items of second sequence are oscillating with the odd terms are two and even terms are zero, so in the two formulas we both adopt $(-1)^{n+1}$ to connect the odd and even of every sequence at last. The above information is obtained by observing, and it is called the method of observation. It is through the observation that we have a preliminary understanding of the problem and

establish the ideas of further problem-solving. Obviously, taking the simple question as an example, the textbook aims to guide students to form a correct way, because often there would not be so simple math problems and the correct problem-solving ideas are more needed. The teacher in the example teaching should focus on the analysis of this kind of example, emphasize the importance of observation, and develop the students' ability of mathematical observation.

2. Mathematical Induction Ability

The so-called induction is to find the relations and differences among different objects on the basis of the observation, and then summarize the characteristics of them have in common, and finally come to the general conclusions. Induction is a kind of inference method ,which is from the individual to the general. Namely, there is a need for us to find out their common part from several things and make a classification and generalization of the points.

Case two: Boyle's law of physics $p = \frac{k}{V}$ (K is a normal number) tells us that for a certain amount of gas, when the volume V is decreased, the pressure P will increase. Try to prove it with the monotonicity of function.

The above case is the first example of the first section C*oncept of Sequence and Simple Representation* on Chapter 2 *The Sequence* of the high school mathematics compulsory textbook-5.This example demands student to use the definition of the functional monotonicity to prove the question. We have a clear understanding of the definition of monotonically increasing or decreasing function by learning the basic knowledge, if for the value of any two independent variables x_1, x_2 within a certain interval D in the domain I , and when $x_1 < x_2$, all have $f(x_1) < f(x_2)$ (or $f(x_1) > f(x_2)$), then we said the function $f(x)$ is increasing function (or decreasing function)on the interval D , but how to write the proof steps regularly as a certification is the key and difficulty of this example. Based on the analysis of this example, teachers should guide student to convert the text to the corresponding symbolic expression step by step, according to the definition of monotonicity. Most importantly , it is necessary to emphasize the induction and reflection after explaining a problem. Taking the simple question as an example, the textbook wants to encourage student to summarize the answers, sum up a more general rule, master the general steps to judge or prove monotonicity of a function with the definition of monotonic function, including “set value”, “calculate poor”, “judge symbol” and “receive conclusion”, thus developing the ability for solving the general problem. The teacher in the example teaching should focus on the analysis of this kind of example, emphasize the importance of induction and summarization in time, develop the students' ability of mathematical induction, help students sort out the context of knowledge, master the connection of knowledge, explicit the rule of knowledge, and build their own "*knowledge system*".

3. Mathematical Association Ability

Association is a kind of think method from one thing to another, and it is also an important way of intuitive thinking. Association generally can be divided into similar, close and contrast Association. Similar Association means the thought of another thing because of the similar external characteristics or similar nature of things; close Association is a thought of space or time with close to things; contrast association is caused by perception or memory of something but they usually have the opposite characteristics.

Case 3: Make a solution set of the inequality $-x^2+2x-3>0$.

The above case is the second example of the second section *One-Variable Quadratic Inequality and its Solution* on Chapter 3 *Inequality* of the high school mathematics compulsory textbook-5.This example requests student to master the general method of solving one-variable quadratic inequality. Under the premise of learning the basic concept of one-variable quadratic inequality and knowing the relationship of one-variable quadratic inequality, one-variable quadratic equations and quadratic function, firstly convert the inequality equivalently to $x^2-2x+3<0$ and find out the converted inequality has a similar form with one-variable quadratic equations and quadratic function, so it is very natural to think about the problem of the root of the one-variable quadratic equations and intersection of quadratic function and coordinate axes X, and finally solve the original inequality within their relationship. In the essence, this example actually wants to convey an important mathematical thinking of association to the students and cultivate the students' ability of mathematical association. There are many reflections of mathematical association in mathematics knowledge, such as the natural association to the exponential when learn the logarithmic function, association to the plane geometry features when learn the solid geometry. The teacher in the example teaching should focus on the analysis of this kind of example, pay attention to the penetration of association thinking, and cultivate the students' ability of mathematical association.

4. Mathematical Conversion Ability

Conversion refers to make a transformation from the problems those to be solved or hard to be solved to the existing knowledge within the scope of the solvable problem, but this transformation should be equivalent, namely the causes and effects of the process are required should be sufficient and necessary, so as to ensure the transformed result is still the result of the original problem.

Case 4: As shown in figure 1.3-6, a shape of frustum of a cone flowerpot's pot mouth's diameter is 20 centimeter, diameter of floor is 15 centimeter, diameter of hole at the bottom for water leakage is 1.5 centimeter, and the basin wall is the length of 15 centimeter. In order to beautify the flowerpot's appearance, it needs to be painted. Known to every square meter with 100 milliliter of paint, then 100 such flowerpot needs how much paint (π is equal to 3.14, the results accurate to 1 milliliter, available to calculator)?

The above case is the second example of the third section *Surface Area and Volume of the Space Geometry* on Chapter 1 *Space Geometry* of the high school mathematics compulsory textbook-2.This example requests student to calculate the surface area of flowerpot according to the characteristics of common geometry, such as the circular cylinder, circular cone and frustum of a cone. Analysis of this example, if we could get a surface area of one flowerpot, then we can simply get the total amount of paint. In addition to, we know the surface area of flowerpot includes the lateral area and bottom area, but excludes the area of the bottom hole. so we can easily solve the problem by knowing the side expansion graph of frustum of a cone is a ring, whose surface area is equal to the upper and lower two floor area and the area of the plus side through the previous study. The purpose of textbook is to request student to master the basic method of transformation to solve the problem of geometric, thus translating a space problem into a plane figure and a complex problem into simple, learning the common methods of solving the problem of geometric transformation, lastly causing the student to form a consciousness of conversion. Conversion is a kind of important mathematics thought, which is common in mathematics. Conversion is widely used in mathematics teaching, such as the conversion of numbers and forms, the conversion of constant and variable, the conversion of general and special ,the equivalent conversion of the three "secondary"

problem, the equivalent conversion of constant establishment problem, the mutual conversion of the distance from the surface to surface, from line to surface ,from point to surface, from point to point in Solid geometry and so on. The thought of conversion can be summarized as: unknown to known, hard to easy, complicate to simple in the process of solving problem, so as to achieve the knowledge migration and solve the problem. The teacher in the example teaching should focus on the penetration of transformation thinking, and cultivate the students' ability of mathematical conversion.

5. Mathematical Modeling Ability

Mathematical modeling is a kind of ability which refine the practical problems in the real world ,abstract as the mathematical model, find the solution of the model, verify the rationality of the model, and use the solutions provided by mathematical model to explain the realistic problems. Mathematical modeling needs solid and optimizing structure of mathematical knowledge, to a certain extent, it reproduces the scientific research process of a miniature.

Case 5:As shown in figure 1.2-1, Assumption that two points A, B are on both sides of the river, there is a need to measure the distance between two points. A person who is on the same side of A selects a point C on his side of the river and measures the distance of AC is 55 meter, $\angle BAC = 51^{o}$, $\angle ACB = 75^{o}$, and now it is demanded to seek the distance between A and B (accurate to 0.1 meter).

The above case is the first example of the second section *Application Examples* on Chapter 1 *The Triangle* of the high school mathematics compulsory textbook-5.This example requests student to figure out the distance between two points on both sides of the river according to the measured data. It is a practical problem in real life without giving the corresponding mathematical model, so it is hard to go on solving the problem only by the data given in the title. Exploration of the example indicates that the key to solve this problem lies in whether or not a appropriate mathematical model can be established. As we can see, the opposite angle of the desired side AB is known and one side of the triangle AC is known in the built triangle model ABC , so we can calculate the opposite side AC ,and finally the side AB can be calculated with the law of sines. The textbook wants to refine a kind of important mathematics thought named mathematical modeling, and make the students form the ability of thinking the practical problems in mathematics. Mathematical modeling is one of the most important skills, and we can easily solve the problems that often seem complex and abstract in our life by establishing appropriate mathematical models. The teacher in example teaching should pay much attention to such examples with real life as the background, start from the student's prior experience in daily life to describe the background of mathematical problems, let students experience the process of abstracting the actual problem into mathematical model and have a further understanding of "*mathematics originate from life, and serve life"*, and promote students' perception of the existence of mathematical model and then cultivate students' ability of mathematical modeling.

6. Conclusion

The examples in mathematics textbook are an essential link between the basic concepts and exercises, and also a key step of knowledge from produce to the application. In the process of teaching, making full use of the textbook examples can consolidate the old knowledge as well as learn new knowledge, so as to strengthen memory, deep understanding, promote knowledge into ability, and lay a foundation for further study .It is necessary to attach great importance to the example teaching and find the essence. Only in this way can the

teachers guide the student to learn, stimulate students' interest, cultivate students' mathematical ability, improve students' mathematical literacy and mathematical spirit.

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