EVALUATE THE PROCESS OF CREATIVE THINKING OF STUDENTS IN CONSTRUCTING MATHEMATICAL PROOF

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Abstract

The purpose of this study is to evaluate creative thinking process of students in constructing mathematical proof on discrete mathematics course. The study examined creativity of students in constructing mathematical proof based on process–based creativity models and referred to as the Osborn and Parnes process model (OPPM). Based on the aims of the study, this research belongs to qualitative study. The data were collected through document student performance results in constructing mathematical proof, interviews, and observations. The Subject of this study were 12 students of mathematics education department in University of Muhammadiyah Tangerang, Banten. The subjects were selected based on their achievement in prior knowledge test. The results this study show that the students construct mathematical proof using the five phase of process–based creativity models. Additionally, this college level course give the students opportunities to explore all creative steps of process–based creativity models.

Key words : creative thinking, process–based creativity models, constructing mathematical proof, discrete mathematics.

1. Introduction

Proof is one of the advanced mathematical skills are perceived as the most difficult ability to achieve the majority of students (Moore, 1994; Weber 2001; Pfeifer 2009; Mujib 2015). Some studies show that the ability of proving students, even students at the college, is still low (Kusnandi 2008; Schwarz & Kaiser, 2009; Pfeifer 2010; Mujib 2015). Many factors affect the ability of evidentiary difficulties of students, one of them is a learning experience. Student experience in preparing a proof in primary schools will have an impact on the ability to prove when they were attending college. As noted by Moore (1994) that one of the reasons why students have difficulty in mathematical proofs was their experience in constructing a mathematical proof, limited to school geometry proof. Correspondingly, based on the results of a study conducted by Sabri (Kusnandi 2008) on the concept of mathematical proof, student teachers suggested that the high school curriculum should prepare students better learning mathematical proof. Weber (2001) found that the initial cause of the failure of students to prove due to the lack of strategic knowledge. While Recio & Godino (2001) found that students' ability to generate a deductive proof is still very limited.

Selden & Selden (Weber, 2003) states that students are not able to determine whether the mathematical proof is valid or not.

The ability to construct mathematical proof and creative thinking is the main asset that should be owned by the students in the study of college-level mathematics, such as calculus, algebraic structures, number theory, probability theory, and discrete mathematics. Discrete mathematics course is a lecture given at the end of the level in Mathematics Education courses that can not escape from studying the mathematical proofs, though better known as discrete mathematics applied mathematics that is very important in our lives. For example the latest smartphone technology applications, network communication systems, and other highly related to discrete mathematics. But before reaching that stage, the concepts related to discrete mathematics theorems need to be proved mathematically. So the ability of proving indispensable in studying this course. And the character of this course discrete, so it requires a certain creativity by students in constructing a mathematical proof. Therefore, the ability to construct mathematical proof on discrete mathematics course have to think of creativity for students. Problems mathematical proofs are part of mathematical problem solving.

Creative problem solving, as a discipline, was first introduced by Alex F. Osborn (1963) and further developed by Parnes (1967) and other members of the Creative Education Foundation (Torrance & safter, 1999). According to the model Osborn and Parnes, creative problem solving occurs in stages as follows: a) Sensitive to the problems and challenges, b) recognition of the problems real (recognizing the real problem), c) generate alternative solutions, d) evaluating the idea, and e) prepare ideas for use.

While Edward de Bono (1970; 1992) introduced the concept of lateral thinking in creative problem solving. The method is distinguished above the target in the dominant ideas and find ways to see the problem before finding an alternative solution (Torrance & safter, 1999). This concept was introduced in the program Cort (Cognitive Research Trust) he develops in which he describes the creative problem solving as a gradual process. The first step involves targeting the dominant issue; The next step involves elaborate thinking that is useful to make a conclusion, summary, key points, and choice. The third step involves making a decision. The fourth step is called "total input" that goes into the thinking. The fifth step includes finding an alternative. Finally, the sixth step includes the implementation of the decision.

Smith (1967) also describes creativity as a process. According to Smith (1967), creativity is usually grown under conditions that facilitated learning. Smith based on the theory that the process of creativity is as important as the product (Clague, 1981). Thus, the process of creative thinking in constructing a mathematical proof is as important as the product of evidence produced by the students. That is, to assess the ability of proof students are not only based on products produced mathematical proof, but the construction process is also equally important mathematical proof.

Therefore, this study aims to answer the problem of how the process of creative thinking of students in constructing a mathematical proof based on the creative process-based models, known as Osborn Parnes Process Model (OPPM).

2. Research Method

Qualitative approach used in this study to observe the students and analyze their work in constructing a mathematical proof into three different settings, which allow gathering information from three different resources. This process has been called by Lincoln and Guba (1985) as triangulation data. The first approach is to observe the activities of students during learning activities in the classroom in constructing a mathematical proof. The second approach involves collecting data during interviews with students and transcript of their statement. The third approach involved assessing the results of their work in constructing a mathematical proof. Analysis of data from all three resources (field notes, interviews, and documents) are based on Miles and Huberman (1994) flow analysis model. The data analysis consists of three concurrent flows of activity: data reduction, data display, and conclusion and verification (Miles & Huberman, 1994).

This research was conducted in six months at the University of Muhammadiyah Tangerang Indonesia. Students involved in this study were students majoring in mathematics education on discrete mathematics courses. Table 1 shows the students who participated in this study. All students who attend classes discrete mathematics consists of 55 students. Students involved in this study were 12 students consisting of four male and eight female.

Table 1. Students Participants

	Gender	
	Males	Females
Students in discrete mathematics course	8	47
Subject in this study	4	8

Twelve students involved in the study were classified based on test results of their prior knowledge, namely high, medium and low as the level of prior knowledge. Each level consists of 4 student ability. Students in the high group in the code with H1, H2, H2 and H4. In the same way to medium level (M1, M2, M3, M4) and a low level (L1, L2, L3, L4).

3. Discussion

When constructing a mathematical proof, the student is given five propositions on discrete mathematics should they prove to the category of easy, medium and hard (see Table 2). The fifth proposition is taken based on the book a discrete Mathematics with Proof by Gosset (2003). Then, the process of creative thinking of students in the analysis based on three sources of data, namely, field observations when they construct the proof, mathematical proof products they produce, and interviews based on observation and mathematical proof products.

Table 2. Five Proposition used in this study

No	Proposition	Category
1	Let $n \in \mathbb{N}$, $n \geq 2$. show that $\binom{2n}{2} = 2\binom{n}{2} + n^2$?	Easy
2	Let $n \in \mathbb{Z}$, $n \geq 1$. Show that $\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \cdots + \binom{n}{n} = 2^{n-1}$	Hard
3	Given set $\{1,2,3,\ldots,10.000\}$. Show that prime number between 1 and 10.000 less than 2288?	Middle
4	Let T be an equilateral triangle of side 1. Prove that if seven points are placed in T, two of them must be distance $\frac{1}{2}$ or less apart ?	Hard
5	Let a, b, c a sequence of consecutive positive integers. Prove that if you add $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$, the numerator of sum (in lowest term) will always be odd ?	Middle

The following will be presented the results of an analysis based on three sources of data:

3.1. Based on Field Observation Findings

Problem awareness and recognition (phase of one and two of Osborn and Parnes model) shown by the students during the observation. Phase one and two, with regard to the sensitivity of the students against the proposition to be proved, looking strategy and facts, and students are aware of the problems faced and to recognize it as a challenge. Based on observations, the second phase of this show by the students. students with high ability, spontaneously they know what and how to prove, it can be seen from their reaction spontaneously say "okay, I know." For the student with the ability to moderate and low. This stage is shown them by reading the questions repeatedly and even there are silent without taking any action.

The third phase of OPPM, idea generation with regard to looking for ideas and possible solutions to construct mathematical proof and involves producing alternative mathematical proof. Based on observations, the third stage is difficult to observe because it relates to what people think college students are not always shown through behavior. For high-level student, looking for ideas and solutions mathematical proof may not need the help of others, believing themselves the focus of constructing a mathematical proof. As for the lower and middle level students, they often ask close friends or teachers to ensure that they get up right idea. Likewise, the fourth and fifth phases, evaluation of ideas and implementation of ideas. This phase involves considerations if the idea gained it correctly, efficiently and practically applied in mathematical problem solving particular mathematical proof. implementation of the ideas will be discussed further in the document which was demonstrated through the implementation of the idea of mathematical proof product.

3.1. Base on Mathematical Proof Product Finding

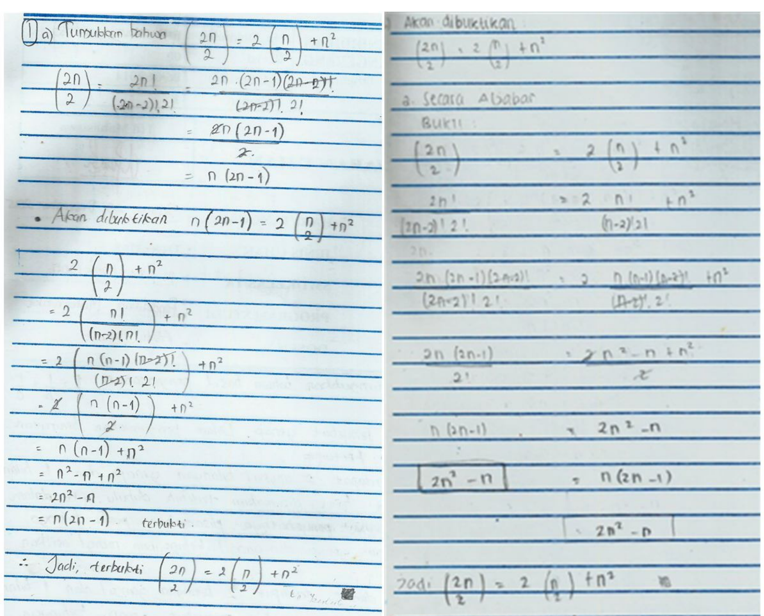
When the student can prove a proposition in Table 2 properly, then the fifth phase of the process of creative thinking by Osborn and Parnes (OPPM) can be reflected in their creative behavior. Table 3 below is a recapitulation of a successful student and failed to prove all five propositions are given:

Table 3. Distribution of mathematical proofs the ability of students based on their prior knowledge.

	Proposition 1		Proposition 2		Proposition 3		Proposition 4		Proposition 5	
	success	Fail	success	fail	Success	fail	success	fail	success	Fail
High	4	0	3	1	4	0	3	1	4	0
Middle	4	0	3	1	4	0	2	2	3	1
Low	3	1	3	1	2	2	2	2	3	1
Total	11	1	9	3	10	2	7	5	10	2
%	91.7	8.3	75.0	25.0	83.3	16.7	58.3	41.7	83.3	16.7

Based on Table 3, most of the students were able to prove all five propositions, a proposition only amounted to 41.7% of four or five students out of 12 students are unable or fail to prove it. Based on preliminary knowledge of students, students who successfully proved the proposition dominated from the top and middle level. This suggests that prior knowledge students have a significant effect on students' ability in constructing a mathematical proof. The process of creative thinking of students by Osborn and Parnes consisting of five phases may appear and is reflected in the creative behavior of students when constructing a mathematical proof. What about the student who fails to construct a mathematical proof, if the fifth stage of the process of creative thinking is reflected in the behavior of their creative in constructing mathematical proof? We will examine in more detail based on mathematical proof of their production.

Figure 1 shows the results of the work of students who succeed and fail in constructing a mathematical proof for the proposition one. Proposition one relating to proof of the equivalence statement $A = B$ that $\binom{2n}{2} = 2\binom{n}{2} + n^2$. Based on the figure 1.a, pattern mathematical proof of the student, the student's first show that $A = C$, then show that $B = C$ and make the conclusion that $A = B$. Students prove the beginning shows that $\binom{2n}{2} = n(2n-1)$ as a new fact which is a process of awareness and recognition of OPPM. From the newly acquired facts, students make the most effective alternative solution by showing that $2\binom{n}{2} + n^2 = 2(2n-1)$ as the process of idea generation. Then the process of making the conclusion that A = B is the stage of evaluation and implementation of ideas.

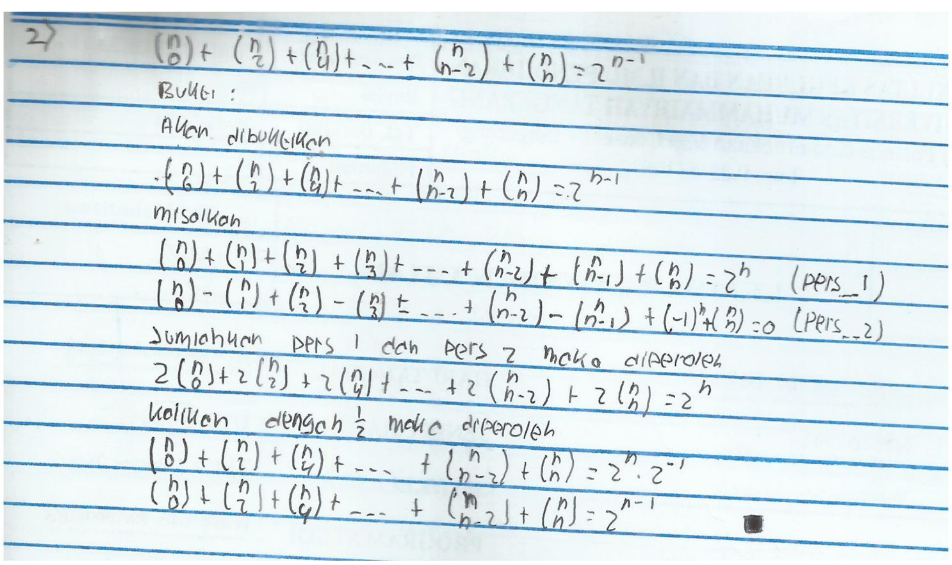


(a) (b)

Figure 1. a) product mathematical proof of successful students b) the product mathematical proof of students who failed

And figure 1.b, students understand what is to be proved that the statement of equivalence $A = B$ and knowing that $A = C$ and $B = C$, namely $\binom{2n}{2} = 2n^2 - n$ dan $2\binom{n}{2} + n^2 = 2n^2 - n$. But, failing in the process of idea generation and implementation of ideas against facts. Failing in strategies evidence, statement of equivalence $A = B$ which will be addressed in use in constructing the proof.

Furthermore, the student's work in constructing the proof of proposition two. Figure 2 is the result of the students who failed in constructing a mathematical proof. The statement "will be proven that $\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \cdots + \binom{n}{n} = 2^{n-1}$ "shows that the students know the purpose of which is to be proved. And the student determines the supporting facts to prove porposisi 2. But the words "supposing" that is used to exploit the fact this is not right. Thus changing the meaning that should a similarity $\binom{n}{0} + \binom{n}{1} + \binom{n}{3} + \cdots + \binom{n}{n} = 2^n$ and $\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \cdots + \binom{n}{n} = 0$ is a similarity it is definitely true that similarity has widened assumed truth value true. If in the view stage process of creative thinking of students by Osborn and Parnes, the stage where the facts relate that there is a series of logical statements that are not met by the student. In other words, the stage of recognition and generate ideas do not emerge when students construct a mathematical proof.


2) $\binom{n}{0}+\binom{n}{2}+\binom{n}{4}+\dots+\binom{n}{n-2}+\binom{n}{n}=2^{n-1}$

Bukti :

Akan dibuktikan

$\binom{n}{0}+\binom{n}{2}+\binom{n}{4}+\dots+\binom{n}{n-2}+\binom{n}{n}=2^{n-1}$

misalkan

$\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\binom{n}{3}+\dots+\binom{n}{n-2}+\binom{n}{n-1}+\binom{n}{n}=2^n$ (pers_1)

$\binom{n}{0}-\binom{n}{1}+\binom{n}{2}-\binom{n}{3}\pm\dots+\binom{n}{n-2}-\binom{n}{n-1}+(-1)^n\binom{n}{n}=0$ (pers_2)

Jumlahkan pers 1 dan pers 2 maka diperoleh

$2\binom{n}{0}+2\binom{n}{2}+2\binom{n}{4}+\dots+2\binom{n}{n-2}+2\binom{n}{n}=2^n$

kalikan dengan $\frac{1}{2}$ maka diperoleh

$\binom{n}{0}+\binom{n}{2}+\binom{n}{4}+\dots+\binom{n}{n-2}+\binom{n}{n}=2^n\cdot 2^{-1}$

$\binom{n}{0}+\binom{n}{2}+\binom{n}{4}+\dots+\binom{n}{n-2}+\binom{n}{n}=2^{n-1}$

Figure 2. The work of students in constructing the proof of proposition 2

Proposition three and five, most of the creative thinking processes of students appeared in constructing a mathematical proof. It can be seen from the presentation students were successful in constructing a mathematical proof. Although there are students who fail, but they can and try to prove a well only when implementing an idea or use a mathematical proof strategies that do not fit.

Most students who failed in constructing a mathematical proof is the fourth proposition. This proposition deals with the pigeons hole principle associated with geometry. The pigeon hole principle is very important to understand when to prove this proposition. In figure 3 it can be seen that the students do not understand this prnciple, allowing him to use the Pythagorean law. Obviously here, stage one and stage two of the process of creative thinking of students failed. As a result, the next stage must fail. Because of this proposition deals with triangles, students tried to relate it to the Pythagorean law and students seem trying to find where it appears the number $\frac{1}{2}$, but failed. Many possible causes students fail to construct mathematical proof, one of them understanding the concepts and understand their problems is very important.

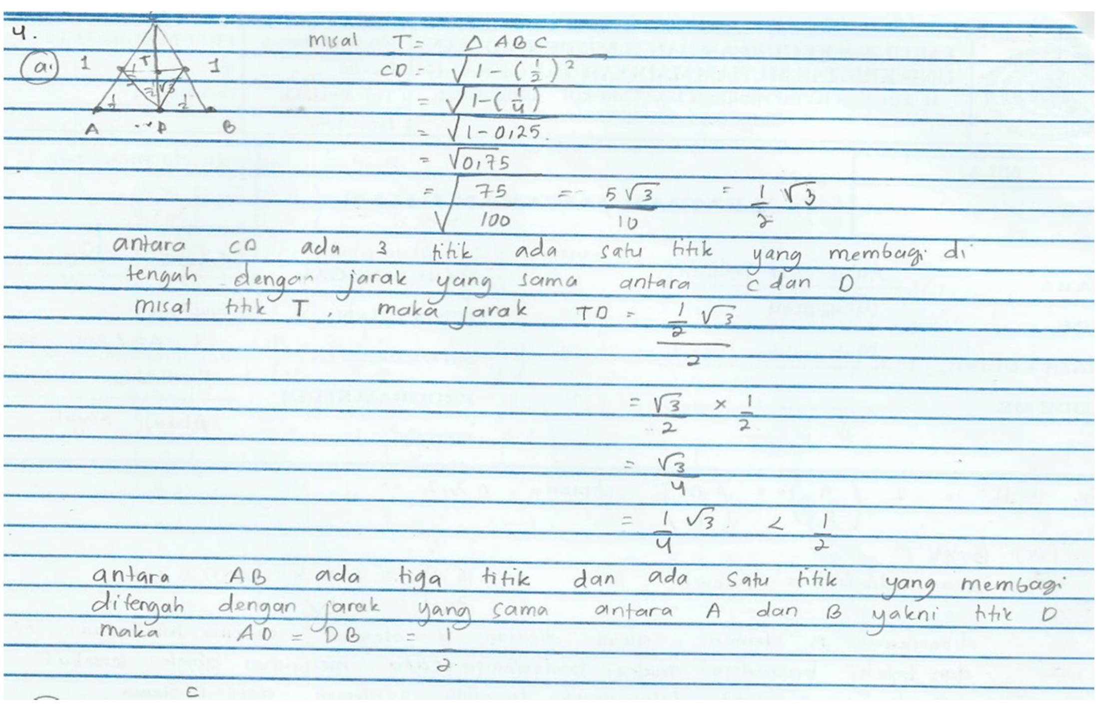


Figure 3. The work of students in constructing the proof of proposition 4

3.2. Based on Interview Findings

The interview was conducted based on the observation and mathematical proof products produced by the students. Interview is used to match and clarify the observation data and student mathematical proof product as a data triangulation. Based on interviews, the fifth phase of OPPM can be passed by students for students with a high level. This means that the process of creative thinking of students in constructing a mathematical proof on discrete mathematics course is reflected in the behavior of their creative. The following is a transcript of the interview with high-ability students:

T: Hello, how are you?
S: Alhamdulllah good
T: What about the exam just now? You could do it right?
S: Of course, I think there is no problem with that.
T: how do you mean? Does it matter less challenging?
S: Not so, sir, because matter is very challenging and I really like the challenge. So, it's no problem for me.
T: So, you can prove to all the questions well?
S: InsyaAllah, I can.
T: Where do you think about that hard to find ideas?
S: Overall, there was no trouble for me to look for ideas. I think ... question no 4. But by thinking calm and focus, a little trial and error I can prove it.
T: how do you believe that the mathematical proof that you get it right?
S: When I finished proving matter, I usually check again by re-reading, whether it is logical or not, whether it is in accordance with the objectives that will be addressed.
T: ok, thanks for your time.
S: ok sir

While in the medium and low level students, they go through five stages OPPM but many still have difficulty in three, four, and five. This shows that they 'understanding the problem and determine the ultimate goal to be in the lead, but failed link between a well known fact that the difficulties in implementing the strategy of constructing a mathematical proof. Therefore, when students fail at an early stage in the process of creative thinking, then this will affect the later stages.

4. Conclusions and Recommendations

Based on the preceding analysis, it can be concluded that the process of creative thinking of students by Osborn and Parnes in constructing mathematical proof on discrete mathematics course is reflected in the behavior of creative students. Based on students' prior knowledge, creative behavior of students consisting of five stages through which students in constructing a mathematical proof that high- and medium-level students. However, for lower level students, the process of creative thinking is not going well.

Thus, in constructing the proof, the experience proves to have an important role. Because, this is related to all students' knowledge of mathematical proof strategy. The ability of proving the highest level in advanced mathematical thinking, so it requires special attention on students' prior knowledge and knowledge of mathematical proof strategy. In addition, technical writing mathematical proof that either might be an interesting study further with regard to proof.

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