Problem-Based Learning to Enhance Mathematical Argumentation Ability of College Students

By

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Abstract

This study aim to find out mathematical argumentation ability enhancement of mathematics education students who take Calculus 1 lesson. The design of this study is quasi experimental, with pretest and posttest control group. Sample selection in this study uses purposive sampling, two classes as experimental class and two classes as control class. Experimental class are taught by using problem-based learning (PBL), and control class are taught by conventional learning (CL). Samples involves 141 students. Instruments which is used is mathematical argumentation ability test. Before implementation of the research, students was given test of prior mathematical knowledge (PMK). Data analysis uses t-test, and one way and two ways ANOVA. Based on data analysis result, it is concluded that there are significant differences on students' mathematical argumentation ability between PMK group (high, mediocre, and low) with PBL approach. The difference of enhancement occur in high PMK group and mediocre PMK group. Significantly, the enhancement of students' mathematical argumentation ability based on PMK group with PBL approach is better compared to students' mathematical argumentation ability which uses CL approach. There is significant enhancement difference of student's mathematical argumentation ability in each PMK group with PBL and CL approach. Concurrently, the two factors of PMK group and learning approach give significant impact towards the enhancement of students' mathematical argumentation ability.

Key Words: *Mathematical argumentation, Problem based-learning*

A. Introduction

The ability in stating a reason which is accompanied with adequate data and theoretical bases from a mathematics problem, whether in the oral or written form is an important part of mathematical ability which must be possessed by students. Reason which is accompanied by correct data and theory will result the correct comprehension towards mathematical concepts. Reason can give explanation about why a question is considered right or wrong. Reason can also change the interpretation towards concept. Such changes are occurred when someone change their comprehension towards several concepts which they use and also the frame of conceptual work, manage and rearrange the framework to accommodate new perspectives.

The ability in stating a mathematics problem can be estimated through students' ability in delivering the idea orally or rewrite the idea in mathematical argumentation. The idea about the optimality of mathematic argumentation ability such as ability in stating reason, data and theoretical

bases, the ability in writing, and the ability in discussing are one of the alternatives in answering the problem. Therefore, it is not impossible that mathematics' subjects and users can find the shape, model and even strategy during working mathematic.

The process of finding solution from a certain problem is not a simple thinking process, in the process of determining a problem solving it requires much thinking skill. The ability in collecting information and data, stating argument, deciding correct theories, determining problem solving steps, are thinking processes which enable students in solving problem.

The argumentation ability is an important ability in learning mathematic. Therefore, mathematic argumentation ability is must be taught to students. Students must acquire argumentation ability in order they are able to solve mathematical problems critically. Argumentation is the essence in the scientific thinking (Cross, 2007).

Argumentation ability is the bases of logical and critical thinking. According to Ennis (1981), critical thinking is the ability in stating reason based on what is believed. Argumentation ability involves stating a reason (critical) accompanied by adequate data and theoretical bases from a mathematical problem (logical).

Argumentation as the bases of critical and logical thinking is still felt hard to be acquired by students (Zeidler, 1997). Von et al (2008), Driver et al (2000), and Newton et al (1999) stated that student's difficulties in stating argumentation is caused by the lack of pedagogical abilities of the teacher in developing argumentation in class. The ability potency in stating critical argument by students is not developed well, because learning process in class. Learning in class usually students are faced with problem situation which must be found its solution. Generally, mathematic learning at college level presents direct problem solving without passing through argumentation process.

Before achieving solving stage, it is required thinking process, data which is known, support from definition or theorem which is used, until it is reached claim stage. Next, it can be looked for the solution, with the expectation that such solution is conducted in directed way.

The argumentation ability is the ability of critical and logical thinking about the relationship between concept and situation. The use of argumentation ability, namely, to explain the relationship between fact, procedure, concept, and solving method which is relates each other. The expectation is, the higher the ability of student's mathematic argumentation ability, the better the ability to state reason from a solution or answer.

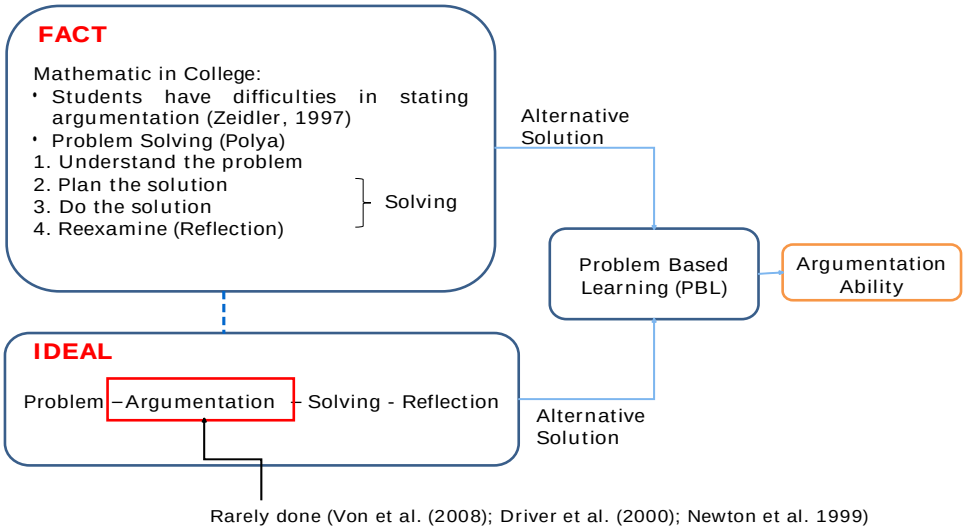


Figure 1. The Frame of Thinking of Students' Mathematic Argumentation Ability Enhancement

Anyway, the teaching and learning process should change to prepare students in overcoming new situation. Student's ability must be enhanced in proposing question, searching and finding the appropriate resources to answer questions, and communicating effectively the solution which they get to others. Problem based learning is one of the approaches in mathematic learning which help students in building critical and logical thinking and skill which is required to communicate successfully today.

Argumentation is the way about how rationally, student answers every question, issues and also denies and overcomes every problem. An argument consists of a claim (solution) which is supported by various principles (guarantee), evidence and various denials towards counter argument which has potency. Developing argumentation in learning environment can enhance problem solving.

Mathematical argumentation is very essential in learning the way to overcome the problem, or as a method to assess ability in solving problem whether for unstructured problem or for structured problem (Jonassen, 2010). According to Nussbaum and Sinatra (2003) that student shows enhancement in problem's thinking, when students can answer correctly or incorrectly and then can state the argument which can give answer correctly and scientifically.

There are many obstacles which are occurred in implementing learning process, for instance, several reasoning aspect which still weak, one of them is argumentation. Student's weakness which is most common in argumentation is the lack of counter argumentation. When a student is asked to state arguments to support or deny something, he usually uses many reasons to support his position (Stein & Bernars, 1999). Qualification and denial is less used in argumentation analysis in mathematics education, but it is proved useful in argumentation analysis which is delivered by students (Inglis et al, 2007).

The effort which is done to find the cause and solution about the lack of students' mathematical argument ability has been studied in many developed countries, by using various educational theories, learning models and approaches which make the knowledge about argumentation broader, Conner (2008) gave the description about the relationship between argumentation with evidence in geometry class. Halpern (2003) explained the process in analyzing student's argumentation description requires steps in analyzing argumentation description, reading and evaluating argumentation description which is based on the relationship between premises, conclusion, assumption, and counter argument.

Through problem based learning it is hoped that students are able to think critically, analyzing complex problems and real world problems, work cooperatively in small groups, skillful in effective communication, accurate orally and written, so it can be seen their mathematic argumentation ability. Mathematical argumentation ability would be much better when students are involved in problem based learning, especially with unstructured problems, and interpretations and also alternative solutions which require argumentation. Students who are demanded to recall the information have few reasons to get involved in argumentation. Problem based learning environment usually presents claims or alternative solutions which must be overcome by students through argumentation.

In terms of pedagogy, the goal of learning today can give wide opportunity to students in doing math. Learning today is more focused on the use of learning environment, among others with PBL. Problem based learning is a learning in class to organize learning around problem based activity, give opportunity for students to deliver their mathematics arguments and ideas, and also communicate to their peers through interaction various components in class towards mathematic learning activities which are presented by teacher. It is in accord with NCTM (2000), which stated

that building challenging and conducive learning environment is an important component from learning.

Learning which provides many mathematic activities opportunity for students in stating argumentation is Problem-Based Learning (PBL), learning which is begun with proposing problem in contextual situation, trending topic and its solving procedure are not defined well. Problem which is presented in the beginning of PBL usually in the form of word problem which is given direction by bringing out counter argument. Counter argument is defined as attributes from well argumentation (Andriessen et al, 2003, Voss et al, 1991) and a standard to asses the argument (Kuhn 1991).

Problem based learning provides many mathematics activities opportunity for students in stating argumentation. Based on characteristic which is possessed by problem based learning, it is hoped that problem based learning can guide students in achieving learning goals, namely, improving students' mathematic argumentation in discussing mathematic problem.

The subject of this study is students on Department of Mathematics Education, with Calculus I lesson. This lesson is used because Calculus I lesson can provide daily mathematic situation problem.

B. Study Method

Population and Sample

The subject in this study is all students who study on Department of Mathematics Education in one of the colleges at Jakarta. The students as subject is based on consideration that Calculus I lesson which is given towards mathematics education freshmen, and students' independency in learning, so it is hoped that the implementation of problem-based learning can be cerried out optimally.

This study sample is the from students of Department of Mathematics Education, which take Calculus I lesson. Sample collected in this study uses purposive sampling, because the sample is taken randomly based on class groups. Students in each class group have similar characteristics, namely, select two classes as control class and two classes as experimental classes from each Department of Mathematics Education. Randomization is conducted by drawing.

Research Procedure

This study is quasi experiment by using problem-based learning. There are two students group which will be examined their mathematical argumentation as the cause from learning treatment which is applied. One group uses problem-based learning (PBL), other group uses conventional learning (CL). From each students group are divided into level category of students' mathematics initial knowledge into above, middle and below class.

There are two stages in this study, namely, first stage (identification and development of learning components) and the second stage is study implementation, in which implementation of all learning series which have been planned.

Instrument and Data Sources

This study uses several instruments: (a) test to measure students' prior mathematical knowledge, (b) test to measure students' mathematical argumentation ability (c) observation sheet during learning and (d) interview, students' reaction towards problem-based learning. Students' mathematical argumentation ability is filtered through written test which is arranged based on three aspects, namely, identifying assumption, identifying relevant and irrelevant data, analyzing argument, answering which is accompanied by reason (clarification), giving reason towards a

conclusion. Mathematical argumentation ability items consist of 8 numbers. This instrument is given to students on pre and post learning.

Data Analysis

Data analyses which are used are quantitative and qualitative analysis. Quantitative analysis consist of descriptive statistic analysis and inferential. The first step is descriptive statistic analysis, such as, average calculation, standard deviation and graphic and also diagram, which is used to see the description in general. To know the enhancement of students' mathematic argumentation ability on both groups, so, it is conducted analysis by using average normalized gain according to Hake (2007).

The second step is inferences statistic analysis which is required as the bases in hypothetical testing, which is begun with normality test and variant homogeneity towards some parts or overall. The next step, to know the difference from each group, there is interaction between free variable with control variable towards bound variable which in accord with hypothesis, the use of one way ANOVA with the assistant of SPSS-19.00 software, with level of reliability is 95%.

C. Study Result

In this part, we will result of the reseacrch. Analysis is conducted towards data, whether there is interaction between learning which is used with the level of prior mathematical knowledge (PMK), and mathematical argumentation ability.

Table 1
Distribution of Study Sample

PMK Group	Experiment (PBL)	Control (CL)	Total
High	18	10	**28**
Mediocre	40	32	**72**
Low	11	30	**41**
Total	**69**	**72**	**141**

Quantitative data is obtained through test of prior mathematical knowledge, and mathematical argumentation ability towards 141 students, which consist of 69 students which is taught by problem-based learning and 72 students which is taught by conventional learning.

Below, is the description of the enhancement of students' mathematical argumentation ability score on pre and post learning.

Table 2
n-Gain Description of Mathematical Argumentation Ability
Based On PMK Learning and Group

PMK	Learning									
	Problem Based Learning (PBL)					Conventional Learning (CL)				
	Min Score	Max Score	Average	SB	n	Min Score	Max Score	Average	SB	n
High	0,456	0,763	**0,618**	0,091	18	0,379	0,574	**0,455**	0,066	10
Mediocre	0,317	0,737	**0,521**	0,102	40	0,121	0,571	**0,377**	0,114	32
Low	0,452	0,770	**0,575**	0,105	11	0,119	0,525	**0,320**	0,110	30
Total	0,317	0,770	**0,555**	0,107	69	0,119	0,574	**0,364**	0,115	72

On Table 2, it can be seen that the enhancement of students' mathematical argumentation ability taught by PBL is much better compared to students who were taught by CL. On that table, average score of mathematical argumentation ability for group who were taught by PBL is 0,555. It means, group which is taught by PBL, get higher score compared with group which is taught by CL, namely, 0,36. Besides, the average enhancement of students' mathematical argumentation ability which is taught by PBL based on PMK group at high, mediocre, and low is higher than students who are taught by CL.

General description about the enhancement of mathematical argumentation ability shows that there are differences between mathematical argumentation ability of students which are taught by PBL compared to students who are taught by CL. Whether the differences significant or no, it depends on statistic test by testing the average score differences.

The calculation of average score differences between students who get PBL and students who get CL are conducted by using t-test. The calculation result is presented below.

To test hypotheses, all requirements have been fulfilled. Hypotheses which are tested are H_0; There are no average score differences on the enhancement of mathematical argumentation ability between students who get PBL and students who get CL. H_a: the average score of enhancement of students' mathematic argumentation ability who get PBL is higher than students who get CL. The criteria of test, if probability value (Sig.) is higher than 0.05, the null hypothesis is accepted.

Table 3
Test of Average Score Differences n-Gain
Mathematical Argumentation Ability of Students
Who are Taught By PBL and Who are Taught by CL

	t-test of Two Average Score Equality			H_0
Argumentation Ability	*t*	df	Sig.	Rejected
	10.174	139	0.000	

On Table 3, it can be seen that null hypotheses is rejected. Therefore, it can be concluded that, enhancement of students' mathematical argumentation ability that get PBL is higher than students who get CL.

Data of n-gain score on students' mathematical argumentation ability overall or based on PMK group which uses PBL approach is distributed normally and its variant is homogenous. To know whether there is differences of average enhancement of students' mathematical argumentation ability based on PMK group and PBL approach, it uses one way ANOVA. The conclusion of ANOVA test is presented on Table 4.

Table 4
ANOVA Average Differences of
Mathematical Argumentation Ability Enhancement
Based On PMK Group and PBL Approach

Source of Differences	Sum of Squares	df	Mean Square	F	Sig.	H_0
Between Groups	0.124	2	0.062	6.222	0.003	Rejected
Within Groups	0.656	66	0.010			
Total	0.779	68				

Based on Table 4, it can be seen that probability value (Sig.) around 0,003. It can be said that, the average enhancement of students' mathematic argumentation ability between PMK group (high, mediocre, and low) which is taught by PBL is significantly different. Next, it is conducted continued difference test, to find the different enhancement on mathematic argumentation ability. Continued test which is used is Scheffe test, the conclusion of the calculation is presented on Table 5.

Table 5
Average Scheffe Test
The Enhancement of Mathematical Argumentation Ability
Based On PMK Group in PBL Approach

(I) PMK Group	(J) PMK Group	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval		H_0
					Lower Bound	Upper Bound	
High	Mediocre	0.098	0.028	0.004	0.027	0.169	Rejected
	Low	0.043	0.038	0.526	-0.0525	0.139	Accepted
Mediocre	High	-0.098	0.028	0.004	-0.169	-0.027	Rejected
	Low	-0.054	0.034	0.286	-0.139	0.031	Accepted
Low	High	-0.043	0.038	0.526	-0.139	0.052	Accepted
	Mediocre	0.054	0.034	0.286	-0.031	0.139	Accepted

Hypotheses which are tested are:

H_0: There are no differences on enhancement of students' mathematical argumentation ability between PMK groups who are taught by PBL approach.

H_a: There are no differences on enhancement of students' mathematical argumentation ability between PMK groups who are taught by PBL approach.

Testing criteria, if probability value (Sig.) higher than $\alpha = 0.05$, so null hypotheses is accepted.

Based on the calculation on Table 5, it seems that probability value (Sig.) for each upper and middle, upper and lower and middle and lower PMK groups. Probability value (Sig.) for enhancement of student's mathematical argumentation ability on upper and middle PMK group less than 0,05. It means null hypothesis is rejected. Therefore, there are significant differences on

student's mathematical argumentation ability between high PMK group and mediocre PMK group. Differed with probability value (Sig.) for PMK group pair of upper and lower and middle and lower, the enhancement of student's mathematic argumentation ability on PMK group more than 0,05. It means that a null hypothesis is accepted. Therefore, it can be concluded that there are no significant differences on enhancement of student's mathematical argumentation ability between upper and lower PMK group and middle and lower PMK group.

It is known that all data on student's mathematical argumentation ability is based on PMK group and learning approach is distributed normally, and its variant is homogenous. To know that there is interaction between learning which uses PMK group on student's mathematical argumentation ability, so, it uses two way ANOVA.

Below is the ANOVA test calculation result which is presented on Table 6.

Table 6
Two Way ANOVA between PMK and Learning Approach towards Enhancement of Students' Mathematical Argumentation Ability

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	H_0
Corrected Model	1.545	5	.309	28.777	.000	
Intercept	24.501	1	24.501	2281.022	.000	
PMK	.159	2	.080	7.402	.001	Rejected
Learning Education	.939	1	.938	87.426	.000	Rejected
PMK*Learning Education	.069	2	.034	3.190	.044	Rejected
Error	1.450	135	.011			
Total	32.538	141				
Corrected Total	2.996	140				

a. R Squared = .516 (Adjusted R Squared = .498)

From the calculation result with ANOVA test on Table 6, is obtained F value for PMK group around 7.402 and probability value (Sig.) around 0,001. It means that a null hypothesis is rejected. Therefore, it can be concluded that PMK group factor gives significant influence towards enhancement of student's mathematical argumentation ability. And so does learning education factor can give significant factor towards the enhancement of student's mathematical argumentation ability. It can be seen by F value for learning approach around 87.426 and probability value (Sig.) around 0.000. This significance value is less than 0,05, so null hypothesis is rejected.

Based on calculation on Table 6, it can be seen that F value for interaction between PMK group and learning approach is 3,190 and probability value (Sig.) is around 0,004. This value is less from significance 0,05, which makes null hypothesis is rejected. Therefore, it can be concluded that PMK group factor and learning approach concurrently can give significant impact towards enhancement of student's mathematical argumentation ability.

To know which PMK group that can give different impact in enhancement of student's mathematical argumentation ability, it can be continued by using Scheffe test. The description of Scheffe test calculation is presented on Table 7.

TABLE 7
The Result of Scheffe Test of the Data Enhancement on Student's Mathematical Argumentation Ability Based On PMK Learning and Group

(I) PMK Group	(J) PMK Group	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval		H_0
					Lower Bound	Upper Bound	
High	Mediocre Low	0,103*	0,023	0,000	0,046	0,160	Rejected
		0,171*	0,025	0,000	0,108	0,234	Rejected
Mediocre	High	-0,103*	0,023	0,000	-0,160	-0,046	Rejected
	Low	0,068*	0,020	0,005	0,018	0,118	Rejected
Low	High	-0,171*	0,025	0,000	-0,234	-0,108	Rejected
	Mediocre	-0,068*	0,020	0,005	-0,118	-0,018	Rejected

*The mean difference is significant at the 0,05 level.

Hypotheses which are tested are:

$H_0 : \mu_1 = \mu_2 = \mu_3$

$H_a : \mu_1 \neq \mu_2 \neq \mu_3$

with:

μ_1 : n-gain average of students' mathematical argumentation ability on high PMK group.

μ_2 : n-gain average of students' mathematical argumentation ability on mediocre PMK group.

μ_3 : n-gain average of students' mathematical argumentation ability on low PMK group.

Testing criteria, if probability value (Sig.) higher than 0,05, so null hypotheses is accepted.

Based on Table 7, it seems that significance value for all data less than 0,05. It shows that students' mathematical argumentation ability on high PMK group is differed significantly compared with mediocre and low PMK group. And so, the improvement of students' mathematic argumentation ability on PMK is differed significantly with students on low PMK group. Average differences (I-J) of enhancement on students' mathematical argumentation ability is positive for each difference between high PMK group until low PMK group. It means, high PMK group give bigger influence towards the enhancement of students' mathematic argumentation compared with mediocre and low PMK group. And so, mediocre PMK group give bigger influence towards the enhancement of students' mathematical argumentation ability compared with low PMK

Next, it is conducted data analysis with assistant of graphic, to see interaction between learning which is used with PMK group towards the enhancement of students' mathematical argumentation ability. With the help of graphic, interaction between PMK groups with learning which is used towards the enhancement of students' mathematical argumentation ability can be seen on Figure 2.

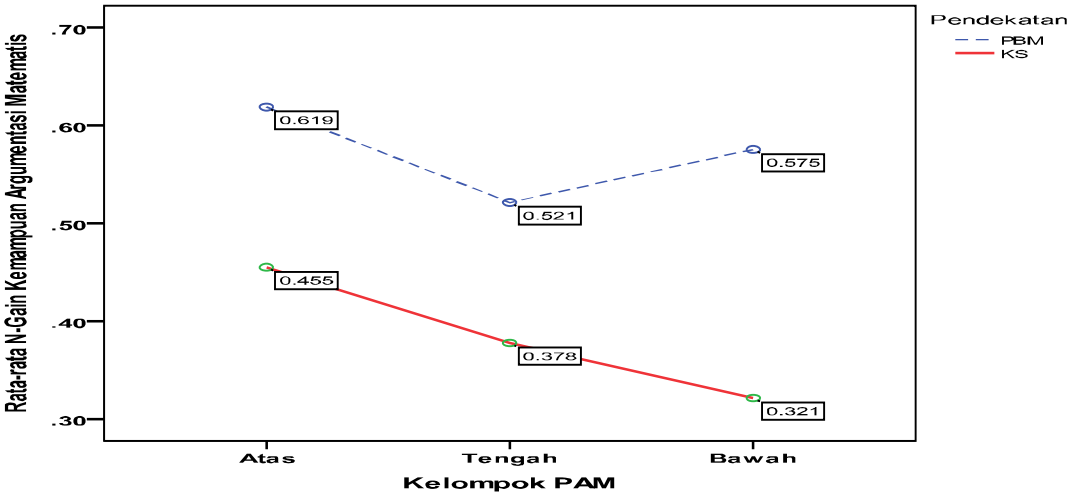


Figure 2. Interaction of Learning Approach and PMK Group in Mathematical Argumentation Ability Enhancement

Based on Figure 2, it seems that there is interaction among learning factor (PBL and CL) with PMK (mediocre and low) on the enhancement of students' mathematical argumentation ability. There is no interaction between learning factor (PBL and CL) with PMK factor (high and mediocre). This can be happened because the differences of the students' mathematical argumentation ability on high PMK group between PBL and CL are differ significantly with the differences on students' mathematical argumentation ability on mediocre PMK group between PBL and CL.

Discussion

The study result shows that there are significant differences on enhancement of mathematical argumentation ability between students who are taught by problem based learning with students who are taught by conventional learning. It can be seen from Table 2. The average of enhancement on students' mathematical argumentation ability who are taught by problem based learning is 0,555 (this improvement is included in medium category) and students who are taught by conventional learning is 0,364 (this improvement is included in medium category). This result is supported by statistic test in which mathematic argumentation ability enhancement of students who are taught by problem based learning (PBL) is better compared to mathematical argumentation ability of students who are taught by conventional learning (CL).

The enhancement of students' mathematical argumentation ability occurred because problem based learning can be a means to practice argumentation skill. Argumentation skill is decided by quantity of practice. So, the more you do the practice, the more skillful you do the argumentation. Osborne (2005) stated that argumentation is a long process which required experience and practice which must be done repeatedly. Besides, the enhancement of students' mathematical argumentation ability occurred because problem based learning can give students the opportunity in understanding basic knowledge, factual and application, which shows communication skill in which effective and accurate whether orally or in written, working cooperatively in small groups (Duch et al, 2001)

The average of mathematical argumentation ability enhancement of students who are taught by PBL for high PMK group is 0,618, mediocre is 0,521, and low is 0,575, each enhancement belong to medium category. Students' mathematical argumentation ability enhancement in mediocre PMK group gets the lowest score, if it is compared with high and low PMK group.

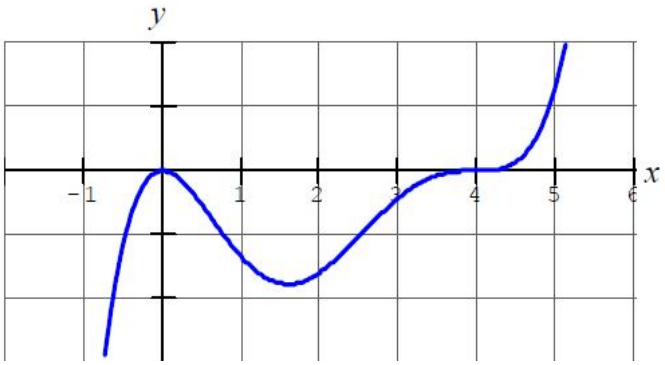
Mathematical argumentation ability enhancement of students who are taught by conventional learning for high PMK group is 0,455, mediocre is 0,377, and lower is 0,320, each of this enhancement belongs to medium category. Mathematical argumentation ability enhancement of students who are taught by conventional learning for high PMK group is better compared to mediocre and lower PMK group.

The students' mathematical argumentation ability enhancement between groups (high, mediocre, and low) who are taught by PBL is significantly different. Significant differences occurred for high and low PMK group. Other finding, namely, the students' mathematical argumentation ability enhancement for mediocre PMK group for students who are taught by PBL is higher from high PMK group for students who are taught by conventional learning. It based on data analysis which results in students' mathematical argumentation ability enhancement on mediocre PMK group for students who are taught by PBL is 0,521 and high PMK group for students who are taught by CL is 0,455. The students' mathematical argumentation ability enhancement on mediocre PMK group for students who are taught by PBL and high PMK group for students who are taught by CL belongs to medium category.

Based on Toulmin model of argumentation level, students' answer towards 7a item is on level 5.

Item number 7a.

Below is f(x) function as follows:

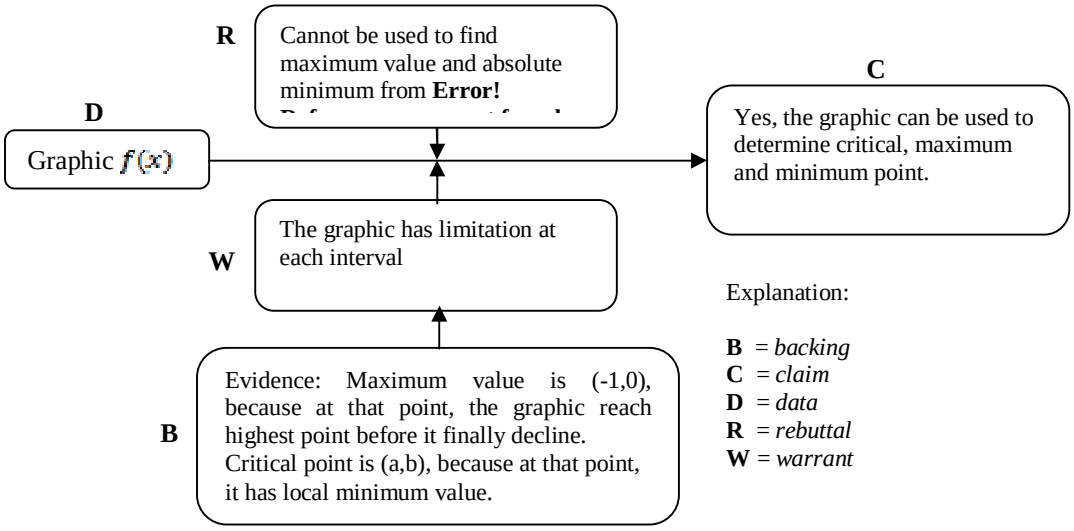


a) Suppose that the graphic which is presented is graphic from f(x). Does such graphic can be used to determine critical, local maximum and minimum, absolute maximum and minimum point from f(x)? Give your answer.

Students are able to understand the minimum and maximum concept from a curve. Students' answer has shown correct way of thinking and has included data, claim, warrant, backing, and rebuttal. Visible data, theoretical backing and warrant which are stated have directed to the claim.

Data : *graphic in item*
Claim : *yes, such graphic can be used to determine critical, maximum and minimum point*
Rebuttal : *but it cannot be used to find absolute maximum and minimum value from f(x)*
Warrant : *on such graphic has limitation on each interval*
Backing : *evidence: Maximum value is (-1,0), because at such point, graphic reaches the highest point before it declines. Critical point is (a,b) point, because at such point has local minimum value*

Visualization which can be given for students' answer as follows:



Based on data analysis, it is obtained that there is interaction between PMK group factor (high, mediocre, and low) and learning factor in students' mathematical argumentation ability enhancement. PMK group factor and learning factor concurrently give significant impact towards enhancement of students' mathematical argumentation ability. This study result shows that a chance to apply problem based learning can be used as alternative learning in enhancing students' mathematical argumentation ability at all level of PMK groups.

The enhancement of students' mathematical argumentation ability between groups (high, mediocre, and low) who are taught by PBL is different significantly. This significant difference occurred at high and mediocre PMK group, while at high and low PMK group is significantly different. It shows to us that problem based learning would be suitable if it is taught to upper PMK group in enhancing students' mathematical argumentation ability. In problem-based learning characteristic, upper PMK group student can optimize communication skill in which effective and accurate, whether verbally or in written. Students who have basic mathematic knowledge (high group), have experience in solving mathematical problem and required knowledge from other groups, so, they can state claim through thinking analysis based on backing with logic evidences and reasons.

D. Conclusion

Overall, it can be concluded that, mathematical argumentation ability enhancement of students who are taught by PBL is better if it is compared with students who are taught by CL. The average of students' mathematical argumentation ability enhancement between PMK group (high, mediocre, and low) who are taught by PBL is different significantly. To be more accurate, schefe test is conducted, which shows that there is significant difference of students' mathematical argumentation ability enhancement between high PMK and mediocre PMK. PMK group factor give significant impact towards enhancement of students' mathematical argumentation ability. And so, the factor of learning approach can give significant impact towards enhancement of students' mathematical argumentation ability. There is also interaction between learning factor and PMK group. It can be seen from students' mathematical argumentation ability score that problem-based learning is more suitable for high and low PMK group of students in enhancing students' mathematical argumentation ability.

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