DEVELOPING TEST OF HIGH ORDER MATHEMATICAL THINKING ABILITY IN INTEGRAL CALCULUS SUBJECT

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Abstract. Learning in Integral Calculus subject is not only memmorizing nor simply applying a given mathematics formula but also requiring high order mathematical thinking ability which will be useful for students. The high order mathematical thinking ability includes mathematical problem solving ability, mathematical reasoning ability, mathematical communication ability, and mathematical connection ability. In order to measure the high order mathematical thinking ability, we need an instrument of high order mathematical thinking test which accommodates the indicators of the abilities. Developing this test aims to measure the improvement of the students' high order mathematical thinking abilities after joining the learning.
Keywords: high order mathematical thinking ability, integral calculus.

1. Introduction

Mathematics is taught at school because mathematics is useful. It is useful for the sake of mathematics itself, and it is useful to solve problem within society (Ruseffendi, 1990). Besides, mathematics is also considered as a tool (Ruseffendi, 1990), thus it can be utilized to serve the other disciplines such as physics, chemistry, and economics. By learning mathematics, students are supposed to possess good ability to face various problems in real world. The fact, however, suggests that the Indonesian score of *Trends in International Mathematics and Science Study* (TIMSS) in 2011 decreased from 399 in 2007 to 392 in 2011 for girls' performance. The boys' performance also decreased from 395 in 2007 to 379 in 2011. The decrease of the score might because of the low ability in high order mathematical thinking of the students. The students' high order mathematical thinking ability does not emerge without design, but it needs to be developed.

Students in mathematics department should have been trained to possess a high order mathematical thinking skill since they are in the first semester of university. One of the subject taught in the first year is intergral calculus. Learning in Integral Calculus subject is not only

memmorizing nor simply applying a given mathematics formula but also requiring high order mathematical thinking ability which will be useful for students (Dwijanto, 2007; Sumarmo, 2005).

The high order mathematical thinking includes the ability of understanding mathematical ideas deeper, observing data and digging the implicit ideas, formulating conjectures, formulating analogy and generalization, reasoning logically, solving problems, communicating mathematically, and relating mathematical ideas to the other intellectual activities. The ability of understanding mathematical ideas deeper, observing data and digging the implicit ideas, formulating conjectures, formulating analogy and generalization, reasoning logically are categorized as mathematical reasoning; while the ability of relating mathematical ideas to the other intellectual activities is categorized as mathematical connection. Thus, in general, the high order mathematical thinking includes mathematical problem solving ability, mathematical reasoning ability, mathematical communication ability, and mathematical connection ability.

2. The High order Mathematical Thinking Ability

Webb and Coxford (as quoted in Sumarmo, 2010) stated that thinking mathematically is categorized into two things, namely the high order mathematical thinking and the lower order mathematics thinking. Doing simple operation, applying direct procedure, working on an algorithm are categorized as the lower order mathematical thinking; while understanding mathematical ideas deeper, observing data and digging the implicit ideas, formulating conjectures, formulating analogy and generalization, reasoning logically, solving problems, communication and connection are categorized as the high order mathematical thinking.

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2.1 Mathematics Problem Solving Ability

Mathematical problem solving ability is the individual capacity to do a series of process having purpose to solve mathematical problem. The mathematical problem solving ability has been the purpose of mathematical learning and is regarded as the core of mathematics (Branca , as quoted

in Abba, 2000:2). If somebody possesses a good problem solving ability, then he possesses a good analysis skill to be applied in various situations. Therefore, the problem solving ability should be internalized since early stage.

Somebody is regarded to possess mathematics problem solving ability if he could: (1) understand problems; (2) choose appropriate strategy to solve the problems; (3) implement the strategy to solve the problems; and (4) re-check the solving process using the chosen strategy (Romberg and Chair, as quoted in Sumarmo, 2005).

2.2 Mathematics Reasoning Ability

Reasoning can be described as a thinking process when somebody tries to show a relation between two things or among several things based on certain regulation which has been proven valid through certain steps and ended by a drawing conclusion (Ramdani, 2011:1). Baroody (1993:58) stated that reasoning is an important tool in mathematics and in daily life, since many problems in mathematics and in daily life require reasoning to solve them.

The indicators of mathematical reasoning in integral calculus subject are as follows: (1) students are able to explain the use of model, facts, properties, and relation; (2) students are able to estimate the solution of the given problem; (3) students are able to use pattern and relation to analyze mathematical situation, drawing analogy and generalization; (4) students are able to execute mathematical calculation based on the agreed procedure; (5) students are able to follow logical reasoning rule; (6) students are able to formulate valid argument and to check the validity of the argument; and (7) students are able to directly or indirectly prove that the conclusion is true (Romberg and Chair, as quoted in Sumarmo, 2005).

2.3 Mathematical Communication Ability

Mathematical communication is an ability to state and to illustrate mathematical ideas into mathematical model or vice versa. The mathematical model could be equation, inequality, notation, picture, or graphic (Sumarmo & Nishitani, 2010).

Within a learning, there is interaction between students and lecturer. The interaction could be oral and written communication. The good teaching and learning interaction could increase the quality of students and lecturer relation, and there will be no gap between them. It is very useful to reach the learning purpose optimally since the learning quality also increase significantly. The interaction which is in a form of communication within mathematical learning aims to bridge

students to the mathematics understanding. Besides, the Minister Regulation Number 22/2006 about the Graduate's Competence Standard states the importance of communication in mathematics learning.

The indicators of the mathematics communication ability in integral calculus subject are as follows: (1) students are able to relate real object, picture, and diagram to mathematics ideas; (2) students ae able to explain mathematics ideas, situation, and relation, verbal or written by using real object, picture, graphic, and algebra; (3) students are able to state daily life situation into mathematical language or symbol; (4) students are able to listen, to discuss, and to write about mathematics; (5) students are able to read a written mathematical representation with understanding; (6) students are able to make conjecture, to formulate argument, to formulate definition and generalization; (7) students are able to explain and to make question about mathematics they learned (Romberg and Chair, as quoted in Sumarmo, 2005).

2.4 Mathematical Connection Ability

According to Ruspiani (2000), mathematical connection ability is individual capacity to connect mathematical concepts within mathematics or to connect mathematical concept to the other field or discipline. This ability is needed because mathematics is a unity, which one concept is related to another. This argument is suitable with Ruseffendi (2006) who states that one of the reasons why students need to exercise with the mathematical connection problems is that mathematical concepts are related each other, such as among theorems, theories, topics, and branches. Thus, in order to make students achieve better in mathematics, students should be given more chances to see the connections.

Sumarmo and Nishitani (2010:6) formulated indicators of mathematical connection ability as follows: (1) looking for relation among representation of concepts and procedures; (2) understanding connection among mathematics topics; (3) applying mathematics in the other fields or in daily life; (4) understanding equivalent representation of a concept; (5) looking for relation among procedures in the equivalent representation; (6) applying relation among topics within mathematics or between mathematical topic with the other topic outside mathematics.

3. Method

This research is a development research which aims to develop test of high order mathematical thinking in integral calculus subject. This research was conducted in mathematics department at a university in Indonesia from May to June 2014.

The research used the R&D steps developed by S. Thagarajan, Dorothy S. Semmel, and Melvyn I. Semmel. It is the 4-D model including the four main steps, namely: (1) define, (2) design, (3) develop, and (4) disseminate. In this research, however, the steps used are only three steps, namely define, design and develop. It because the aims of this research is to produce test which is revised based on expert recommendation and followed by testing in order to know the validity, reliability, discriminating power, and level of difficulty of the test items.

4. Result and Discussion

The development of the high order mathematics thinking test in this research was done following these steps.

4.1 Define

In this step, we analyzed the curriculum and the abilities which will be developed. The 2013 curriculum demands assessment which does not only concern in the final result but also the process. Besides, the mathematics ability such as problem solving, reasoning, communication, and connection also should be developed. Further, we also analyzed the material in the integral calculus subject and its application.

4.2 Design

In this step, we determine the format of the test and the purpose of the test. Since the test aims to measure the improvement of the high order mathematics thinking ability, then we use essay format. We also determine that the allocation time to work on the test is 150 minutes.

The high order mathematics thinking test was built based on the material in integral calculus subject which has been adjusted with the measured abilities. Based on the consideration above, we built reference of the test containing 18 indicators and was then formulated into the high order mathematics thinking test items.

4.3 Develop

4.3.1 Expert Validation

Before it is tested in limited scale and large scale, the items were given to experts to be validated. The experts assessed the content validation, face validation, and construct validation. The experts involved were a professor in mathematics education evaluation, a professor in mathematics education, a doctorate of mathematics education, and two masters of mathematics education who are in the middle of doctorate program. The result of the validation showed that all the experts agreed that 100% of the test items are valid for content validity, face validity, and construct validity.

4.3.2 Limited Testing

After being validated by experts, the test was tested limitedly toward 5 students who have passed the integral calculus subject. The aim of the testing was to know the readability and whether what the problem wants is understood by the students. Based on the limited testing result, the high order mathematical thinking test can be understood well by the students thus it is appropriate to be used for research.

4.3.3 Large Scale Testing

The large scale testing was conducted by giving the test to 36 students who have passed the integral calculus subject. This testing aims to determine the reliability, validity, discriminating power, and level of difficulty of the test items. The result is as follows:

Based on the output of Software IBM SPSS 20 we gained result of $r_{11} = 0{,}912$, thus the test items of high order mathematical thinking ability have high reliability (Suherman, 2003).

Based on the output of Software IBM SPSS 20 we gained result of validity that only item number 1 is considered not valid. There are 11 items considered having intermediate validity. They are items number 2a, 3c, 3d, 5a, 5b, 5c, 5d, 6, 7, 8b, and 8c. And there are 6 items having strong validity. They are items number 2b, 3a, 3b, 3e, 4, and 8a.

Based on the calculation using Software Microsoft Excel 2007 we gained reslt that there are 1 item having bad discriminating power. It is the item number 1. There are 5 items having average discriminating power. They are items number 3c, 5c, 6, 8a, and 8b. And there are 12 items having good discriminating power. They are items number 2a, 2b, 3a, 3b, 3d, 3e, 4, 5a, 5b, 5c, 5d, 6, 7, and 8c.

The calculation of difficulty level by using Software Microsoft Excel 2007 gained result that there are 6 items are easy. They are items number 1, 3c, 5c, 6, 8a, and 8b. There are 11 items having average level of difficulty. They are items number 2a, 2b, 3a, 3b, 3d, 3e, 4, 5a, 5b, 5d, and 7. And 1 item is difficult, which is item number 8c.

Based on he calculation above, we know that item number 1 could not be used for the research, while the rest of the items are appropriate.

5. Comclusion

This research develops test of high order mathematical thinking ability by using the R&D steps developed by S. Thagarajan, Dorothy S. Semmel, and Melvyn I. Semmel. It is the 4-D model including the four main steps, namely: (1) define, (2) design, (3) develop, and (4) disseminate. In this research, however, the steps used are only three steps, namely define, design and develop. It because the aims of this research is to produce test which is revised based on expert recommendation and followed by testing in order to know the validity, reliability, discriminating power, and level of difficulty of the test items.

The test of high order mathematical thinking ability in integral calculus subject developed in this research could be considered valid and reliable. Thus the test could be used to measure the improvement of the high order mathematical thinking ability of the students joining the subject.

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